Day 296 **IGCSE 10-a-day: Model Solutions**

1 Matrix times vector and order

$A = \begin{pmatrix} -4 & 1 \\ -2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

(a) Write down the order of matrix A.

(b) Work out AB.

(a) 2 rows and 2 columns: 2 × 2

(b) $AB = \begin{pmatrix} (-4) \times 5 + 1 \times 3 \\ (-2) \times 5 + 0 \times 3 \end{pmatrix}$

$AB = \begin{pmatrix} -17 \\ -10 \end{pmatrix}$

Answer: (a) 2 × 2 (b) $\begin{pmatrix} \mathbf{-17} \\ \mathbf{-10} \end{pmatrix}$

2 Tangent where the curve crosses the y-axis

The curve $y = x^3 + x^2 - 2x + 2$ crosses the y-axis at the point A.

The tangent to the curve at A meets the x-axis at B.

Find the coordinates of B.

At A, x = 0 and y = 2

$\frac{dy}{dx} = 3x^2 + 2x - 2$, so the gradient at A is −2

Tangent: $y = -2x + 2$

At B, y = 0: $-2x + 2 = 0$, so x = 1

Answer: (1, 0)

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 33°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 33°

Angle COB is an exterior angle of triangle OAC: 33 + 33 = 66°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 66°

4 Increasing for x greater than k

The function $f(x) = x^2 + 2x + 4$ is increasing when x > k.

Find the value of k.

$f'(x) = 2x + 2$

The turning point is where $f'(x) = 0$: $2x + 2 = 0$

$x = -1$

$f'(x)$ is positive for $x > -1$, so $k = -1$

Answer: k = −1

5 Depreciation

A laptop is bought for £8,000.

Its value falls by 25% each year.

Work out its value after 3 years. (Calculator)

Multiplier = 1 − 25 ÷ 100 = 0.75

Value = 8000×0.75^3

= £3,375

Answer: £3,375

6 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is $y = -x + 17$.
(a) Estimate y when x = 9.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = -1 \times 9 + 17 = 8$
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 8 (b) x = 15 is outside the range of the data

7 Inverse functions

$f(x) = \frac{8}{x-3} - 1$
(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(-2)$.

(a) Let $y = \frac{8}{x-3} - 1$, so $y + 1 = \frac{8}{x-3}$
Make x the subject: $x - 3 = \frac{8}{y+1}$
$x = \frac{8}{y+1} + 3$
$f^{-1}(x) = \frac{8}{x+1} + 3$
(b) $f^{-1}(-2) = \frac{8}{-2-(-1)} + 3 = -5$
Answer: (a) $f^{-1}(x) = \frac{8}{x+1} + 3$ (b) -5

8 Fractional power functions

$f(x) = 4x^{1/2} - 6x^{-1/2}$
Find $f'(x)$ and work out the value of $f'(1)$.

$4x^{1/2} \to 2x^{-1/2}$, $-6x^{-1/2} \to 3x^{-3/2}$
$f'(x) = 2x^{-1/2} + 3x^{-3/2}$
$f'(1)$: $\sqrt{1} = 1$, so $f'(1) = 5$
Answer: $f'(x) = 2x^{-1/2} + 3x^{-3/2}$, $f'(1) = 5$

9 Trigonometric equations

Solve $4 \sin x° + 7 = 3$ for $0 \leq x \leq 360$

Rearrange: $4 \sin x° = -4$, so $\sin x° = -1$
From the graph of $y = \sin x°$, $\sin x° = -1$ when x = 270
Answer: x = 270

10 Surds and fractions

Simplify $\frac{3}{3+\sqrt{3}} + \frac{1}{3-\sqrt{3}}$
Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{3})(3-\sqrt{3}) = 9 - 3 = 6$
Numerator: $3(3-\sqrt{3}) + (3+\sqrt{3})$
$= 12 - 2\sqrt{3}$
$\frac{12-2\sqrt{3}}{6} = \frac{6-\sqrt{3}}{3}$
Answer: $\frac{6-\sqrt{3}}{3}$