Day 295 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = -x^5 - 6x^4 + 2x^2$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$-x^5 \to -5x^4$, $-6x^4 \to -24x^3$, $2x^2 \to 4x$

$\frac{dy}{dx} = -5x^4 - 24x^3 + 4x$

Answer: $\mathbf{\frac{dy}{dx} = -5x^4 - 24x^3 + 4x}$

2 Quadratic sequences

Here are the first five terms of a quadratic sequence.
13, 21, 33, 49, 69
Find an expression for the nth term.

First differences: 8, 12, 16, 20
Second differences: 4, 4, 4, so $a = 4 \div 2 = 2$
Subtract $2n^2$ from each term: 11, 13, 15, 17, 19
The remaining linear sequence has nth term $2n + 9$
nth term $= 2n^2 + 2n + 9$
Answer: $\mathbf{2n^2 + 2n + 9}$

3 Increasing and decreasing with powers

$f(x) = x^{\frac{3}{2}} - 9x$ for $x > 0$
(a) Find the range of values of x for which f is a decreasing function.
(b) Find the least value of f(x).

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 9$

$f'(x) < 0$ when $\frac{3}{2}x^{\frac{1}{2}} < 9$, so $x^{\frac{1}{2}} < 6$

Square both sides: $x < 36$, so $0 < x < 36$

f decreases then increases, so the least value is at $x = 36$: $f = 36^{\frac{3}{2}} - 9 \times 36 = 216 - 324 = -108$
Answer: (a) $\mathbf{0 < x < 36}$ **(b)** $\mathbf{-108}$

4 Inverse functions

$f(x) = \frac{8}{x - 3} + 5$
(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(3)$.

(a) Let $y = \frac{8}{x - 3} + 5$, so $y - 5 = \frac{8}{x - 3}$

Make x the subject: $x - 3 = \frac{8}{y - 5}$

$x = \frac{8}{y - 5} + 3$

$f^{-1}(x) = \frac{8}{x - 5} + 3$

(b) $f^{-1}(3) = \frac{8}{3 - 5} + 3 = -1$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{8}{x - 5} + 3}$ **(b)** $\mathbf{-1}$

5 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 3 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 6 cm

Area of the square = $\frac{1}{2}$ × diagonal × diagonal = $\frac{1}{2}$ × 6 × 6 = 18

Area of circle = $\pi \times 3^2 = 9\pi$

Shaded area = $9\pi - 18$ cm^2

Answer: $(9\pi - 18)$ cm^2

6 Solve f′(x) = 0 with a reciprocal term

$f(x) = 4x^3 + \frac{7500}{x}$

Find the values of x for which f′(x) = 0.

$f'(x) = 12x^2 - \frac{7500}{x^2}$

$f'(x) = 0$: $12x^2 = \frac{7500}{x^2}$

$x^4 = 625$
$x = 5$ or $x = -5$
Answer: x = 5 or x = −5

7 Percentages of percentages

In a group of visitors, 35% are adults and the rest are children.
50% of the adults and 30% of the children buy a souvenir.
What percentage of all the visitors buy a souvenir?

Take 100 visitors: 35 are adults and 65 are children
50% of 35 = 17.5 and 30% of 65 = 19.5
17.5 + 19.5 = 37 out of 100
Answer: 37%

8 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 14 cm, BC = 6 cm, CD = 8 cm,
angle ABC = 127° and angle ACD = 85°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 14^2 + 6^2 - 2 \times 14 \times 6 \times \cos 127°$ so AC = 18.251 cm
Area ABC = ½ × 14 × 6 × sin 127° = 33.54
Area ACD = ½ × 18.251 × 8 × sin 85° = 72.73
Total area = 33.54 + 72.73 = 106
Answer: 106 cm^2

9 Ratio and mixtures

Bottle A holds concentrate and water in the ratio 4 : 6.
Bottle B holds concentrate and water in the ratio 3 : 5.
20 litres from bottle A and 8 litres from bottle B are mixed.
Work out the ratio of concentrate to water in the mixture.

From A: concentrate = 20 × $\frac{2}{5}$ = 8 litres, water = 12 litres

From B: concentrate = 8 × $\frac{3}{8}$ = 3 litres, water = 5 litres

concentrate: 8 + 3 = 11 and water: 12 + 5 = 17
11 : 17
Answer: 11 : 17

10 Probability and quadratic equations

A bag contains x green discs and 5 white discs.
Two discs are taken at random without replacement.

The probability that both are green is $\frac{1}{6}$.

(a) Show that $x^2 - 3x - 4 = 0$.
(b) Hence work out the number of discs in the bag.

(a) P(both green) = $\frac{x}{x+5} \times \frac{x-1}{x+4} = \frac{1}{6}$

$6\,x(x-1) = 1(x+5)(x+4)$

$6x^2 - 6x = x^2 + 9x + 20$ so $5x^2 - 15x - 20 = 0$; divide by 5: $x^2 - 3x - 4 = 0$

(b) Solve: $x = 4$ or $x = -1$

x cannot be negative, so $x = 4$

Number of discs = 4 + 5 = 9

Answer: x = 4, so 9 discs in the bag