**Day 294** **IGCSE 10-a-day: Model Solutions**

## 1 Inequalities

(a) Solve $3 - 2x \geq 9$
(b) n is an integer. Write down all the values of n that satisfy
$-2 \leq 2n < 4$

(a) Subtract 3: $-2x \geq 6$
Divide by $-2$ and reverse the inequality sign: $x \leq -3$
(b) Subtract 0: $-2 \leq 2n < 4$
Divide by 2: $-1 \leq n < 2$
Integers: $-1, 0, 1$
**Answer: (a) $x \leq -3$ (b) $-1, 0, 1$**

## 2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(5x - 20)°$
Angle PQR = $(2x + 5)°$
Angle PRS = $(5x + 17)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$5x + 17 = 7x - 15$
$2x = 32$
$x = 16$
(b) Angle PRS = 97° so angle PRQ = 180° − 97° = 83°
**Answer: (a) x = 16 (b) 83°**

## 3 Increasing for x greater than k

The function $f(x) = 3x^2 - 30x - 7$ is increasing when $x > k$.
Find the value of k.

$f'(x) = 6x - 30$
The turning point is where $f'(x) = 0$: $6x - 30 = 0$
$x = 5$
$f'(x)$ is positive for $x > 5$, so $k = 5$
**Answer: k = 5**

## 4 Error intervals

The length of a rope is 0.07 m, correct to 1 significant figure.
Write down the error interval for the length of a rope, using l.

The value has been rounded to the nearest 0.01
Half of this is 0.005, so go 0.005 below and 0.005 above
0.07 − 0.005 = 0.065 and 0.07 + 0.005 = 0.075
$0.065 \leq l < 0.075$
**Answer: $0.065 \leq l < 0.075$**

## 5 Density of an alloy

An alloy is made by mixing 80 $cm^3$ of nickel with 80 $cm^3$ of lead.
The density of nickel is 8.9 $g/cm^3$. The density of lead is 11.3 $g/cm^3$.
Work out the density of the alloy.

Mass of nickel = 80 × 8.9 = 712 g
Mass of lead = 80 × 11.3 = 904 g
Total mass = 1616 g, total volume = 160 $cm^3$
Density = 1616 ÷ 160 = 10.1 $g/cm^3$
**Answer: 10.1 $g/cm^3$**

## 6 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by

$N = -2t^3 + 24t^2 + 8t + 1000$ for $0 \leq t \leq 16$

Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -6t^2 + 48t + 8$

Rate is greatest when $\frac{d^2N}{dt^2} = -12t + 48 = 0$, so $t = 4$

$\frac{d^2N}{dt^2}$ is decreasing (gradient $-12$), so the rate is a maximum

$\frac{dN}{dt} = 104$ when $t = 4$

**Answer: t = 4, rate 104 per hour**

## 7 Quadratic inequalities

Solve $x^2 + 2x < 8$

Rearrange: $x^2 + 2x - 8 < 0$

Factorise: $(x + 4)(x - 2) < 0$

Critical values: $x = -4$ and $x = 2$

Sketch the U-shaped curve: the expression is negative between the critical values

**Answer: $-4 < x < 2$**

## 8 Bounds of roots

The volume of a cube is 718 $cm^3$, correct to the nearest $cm^3$.

Work out the upper bound of the edge length of the cube.

Give your answer correct to 3 significant figures. (Calculator)

$717.5 \leq$ volume $< 718.5$

Upper bound of edge $= \sqrt[3]{718.5}$

$= 8.95658... = 8.96$ (3 s.f.)

**Answer: 8.96 cm**

## 9 Bounds: finding an unknown length

A rectangle has area 56 $cm^2$, correct to 2 significant figures.

Its length is 8.300000000000000710542735760100185871124267578125 cm, correct to 2 significant figures.

Work out the lower bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: $55.5 \leq A < 56.5$. Length: $8.250000000000000710542735760100185871124267578125 \leq l < 8.350000000000000710542735760100185871124267578125$

LB width = LB area ÷ UB length = $55.5 \div 8.350000000000000710542735760100185871124267578125 = 6.65$

**Answer: 6.65 cm**

## 10 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + x - 2$ at the point P where $x = -1$ meets the curve again at Q.

Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 1$: at P, gradient $= -1$ and $y = -2$

Normal: $y = x - 1$

Intersection: $x^2 - 1 = 0$

One root is $x = -1$, and the sum of the roots is 0

So $x_Q = 0 - (-1) = 1$

$y = 0$

**Answer: (1, 0)**