Day 293 **IGCSE 10-a-day: Model Solutions**

1 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -5t^2 + 40t$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 9. State whether it is moving up or down.

$\frac{dh}{dt} = -10t + 40 = 0$ gives $t = 4$

$\frac{d^2h}{dt^2} = -10 < 0$ so maximum. $h = 80$

When $t = 9$: $\frac{dh}{dt} = 40 - 10 \times 9 = -50$

Answer: (a) 80 m (b) −50 m/s, moving down

2 Equations with fractions

Solve $\frac{x-2}{5} + \frac{x+4}{6} = 1$

Multiply every term by 30: $6(x - 2) + 5(x + 4) = 30$

Expand: $11x + 8 = 30$

$11x = 22$

$x = 2$

Answer: x = 2

3 Equation of a normal

A curve has equation $y = -x^2 - 3x - 6$.

Find the equation of the normal to the curve at the point where x = 0.

When $x = 0$, $y = -6$ and $\frac{dy}{dx} = -3$

Gradient of the normal = $-1 \div (-3) = \frac{1}{3}$

$y + 6 = \frac{1}{3}x$

$y = \frac{1}{3}x - 6$

Answer: $y = \frac{1}{3}x - 6$

4 Percentage change

The mass of a sample is increased by 100%.

By what percentage must the new value be decreased to get back to the original value?

Let the original be 100. After a 100% increase it is 200.

To return to 100, decrease by 100

Percentage decrease = $\frac{100}{200} \times 100 = 50\%$

Answer: 50%

5 Pressure

A concrete slab in the shape of a cuboid weighs 660 N.

Its smallest face has area 150 cm^2. Its largest face has area 800 cm^2.

Pressure = force ÷ area

Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 800 cm^2 = 0.08 m^2

(a) Greatest pressure is on the smallest face: $660 \div 0.015 = 44{,}000$ N/m^2

(b) Least pressure is on the largest face: $660 \div 0.08 = 8{,}250$ N/m^2

Answer: (a) 44,000 N/m^2 (b) 8,250 N/m^2

6 Probability with sets

n(ξ) = 120, n(A) = 37, n(B) = 44 and n(A ∩ B) = 29.
Find P(A | B) and P(B | A′).

$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{29}{44}$

n(A′) = 120 − 37 = 83 and n(B ∩ A′) = 44 − 29 = 15

$P(B \mid A') = \frac{15}{83}$

Answer: $P(A \mid B) = \frac{29}{44}$, $P(B \mid A') = \frac{15}{83}$

7 Divide then differentiate

$y = \frac{-3x^4 - 6x^2 + 3}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = -3x^3 - 6x + 3x^{-1}$

$-3x^3 \to -9x^2$, $-6x \to -6$, $3x^{-1} \to -3x^{-2}$

$\frac{dy}{dx} = -9x^2 - 6 - 3x^{-2}$

Answer: $\frac{dy}{dx} = -9x^2 - 6 - \frac{3}{x^2}$

8 Mixtures and percentage profit

A shop mixes two types of mixed nuts.
Type A costs £6.00 per kg and type B costs £12.00 per kg.
They are mixed in the ratio A : B = 2 : 4.
The mixture is sold for £18.00 per kg.
Work out the percentage profit.

Cost of 6 kg of mixture = 2 × £6.00 + 4 × £12.00 = £60.00
Cost per kg = £60.00 ÷ 6 = £10.00
Profit per kg = £18.00 − £10.00 = £8.00
Percentage profit = 8.00 ÷ 10.00 × 100 = 80%
Answer: 80%

9 Bearings with the sine and cosine rules

A plane leaves an airport P and travels 104 km on a bearing of 145° to Q. (Calculator)
It then travels 75 km on a bearing of 070° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 145° + 180° = 325°
Angle PQR = 105° (the difference between the bearings 325° and 070°)
$PR^2 = 104^2 + 75^2 - 2 \times 104 \times 75 \times \cos 105° = 20479$, so PR = 143.1 km

(b) $\cos(QPR) = \frac{104^2 + PR^2 - 75^2}{2 \times 104 \times PR}$, so angle QPR = 30.4°
R is anticlockwise of the direction PQ, so the bearing is 145° − 30° = 115°
Answer: (a) 143.1 km (b) 115°

10 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $4n^2 + 6n - 5$.

(a) Show that the difference between the (n + 1)th term and the nth term is 8n + 10.

(b) Two consecutive terms of the sequence differ by 146. Find the value of n for the first of these two terms.

(a) (n + 1)th term = $4(n + 1)^2 + 6(n + 1) - 5 = 4n^2 + 14n + 5$

Subtract the nth term: $4n^2 + 14n + 5 - (4n^2 + 6n - 5) = 8n + 10$

(b) $8n + 10 = 146$ so $8n = 136$

$n = 17$

Answer: (b) n = 17