Day 292 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $x^2 - 3yz$
when x = 3, y = 3 and z = 2.

Substitute the values: $3^2 - 3 \times 3 \times 2$
$= 9 - 18$
$= -9$
Answer: -9

2 Gradient with fractional or negative powers

The equation of a curve is $y = 2x^{3/2} + 5x - 2\sqrt{x}$.
Work out the gradient of the curve at the point where x = 16.

$\frac{dy}{dx} = 3\sqrt{x} + 5 - x^{-1/2}$

When x = 16: gradient = $\frac{67}{4}$

Answer: $\frac{67}{4}$

3 Ratio and algebra

In a fruit box, the ratio of the number of apples to the number of pears is 4 : 5.
Then 2 apples are removed and 7 pears are removed.
The ratio is now 7 : 8.
How many pears were there at the start?

Let the numbers at the start be 4k and 5k
$(4k - 2) : (5k - 7) = 7 : 8$, so $8(4k - 2) = 7(5k - 7)$
$32k - 16 = 35k - 49$
$-3k = -33$, so k = 11
Number of pears at the start: $5 \times 11 = 55$
Answer: 55

4 Divide then differentiate

$y = \frac{-2x^3 + 2x^2 + 1}{x^2}$

Find $\frac{dy}{dx}$.

Divide each term by x^2: $y = -2x + 2 + x^{-2}$
$-2x \rightarrow -2$, $x^{-2} \rightarrow -2x^{-3}$
$\frac{dy}{dx} = -2 - 2x^{-3}$

Answer: $\frac{dy}{dx} = -2 - \frac{2}{x^3}$

5 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (4x)°, angle BCD = (3x + 26)° and angle ABC = (2x + 30)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
$(4x) + (3x + 26) = 180$
$7x + 26 = 180$, so $7x = 154$
$x = 22$
(b) Angle ABC = $2 \times 22 + 30 = 74°$
Angle ADC = $180 - 74 = 106°$
Answer: (a) x = 22 (b) 106°

6 Standard form problems

A factory makes 5.2×10^3 bottles each day.
It runs for 2.5×10^1 days.
How many bottles does it make?
Give your answer in standard form.

Total = $(5.2 \times 10^3) \times (2.5 \times 10^1)$
$5.2 \times 2.5 = 13$ and $10^3 \times 10^1 = 10^4$
$= 1.3 \times 10^5$

Answer: 1.3×10^5

7 Inequalities from a region

Region R is shaded. Solid boundaries are included, dashed ones are not.
(a) Write down the three inequalities that define R.
(b) How many points with integer coordinates lie in R?

The vertical line is $x = 3$, the horizontal line is $y = 1$
The sloping line passes through (9, 1) and (3, 6): $5x + 6y = 51$
R is to the right of $x = 3$, above $y = 1$ and above the sloping line. Solid lines are included, dashed lines are not:
$x \geq 3$, $y \geq 1$, $5x + 6y \leq 51$
Count the integer points column by column: $6 + 5 + 4 + 3 + 2 + 1 + 1 = 22$
Answer: (a) $x \geq 3$, $y \geq 1$, $5x + 6y \leq 51$ (b) 22

8 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by
$N = -2t^3 + 12t^2 + 4t + 200$ for $0 \leq t \leq 8$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -6t^2 + 24t + 4$

Rate is greatest when $\frac{d^2N}{dt^2} = -12t + 24 = 0$, so $t = 2$

$\frac{d^2N}{dt^2}$ is decreasing (gradient -12), so the rate is a maximum

$\frac{dN}{dt} = 28$ when $t = 2$

Answer: t = 2, rate 28 per hour

9 Functions that commute

$f(x) = 3x - 6$ and $g(x) = kx + 6$, where k is a constant.
$fg(x) = gf(x)$ for all values of x.
Find the value of k.

$fg(x) = 3(kx + 6) - 6 = 3kx + 12$
$gf(x) = k(3x - 6) + 6 = 3kx - 6k + 6$
Equate the constant terms: $12 = -6k + 6$
$-6k = 6$, so $k = -1$
Answer: $k = -1$

10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on OB with ON : NB = 1 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{5}\mathbf{a} = \frac{1}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{2}t\mathbf{b}$

Equate: $\frac{1}{5}s = 1 - t$ and $1 - s = \frac{1}{2}t$

Solving: $s = \frac{5}{9}$ and $t = \frac{8}{9}$

OX = $\frac{1}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$

Answer: (a) $\frac{1}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$