Day 291 **IGCSE 10-a-day: Model Solutions**

1 Arc length and sector area

A sector of a circle has radius 15 cm.

The length of the arc of the sector is $\frac{25}{2}\pi$ cm.

(a) Work out the angle of the sector.
(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 15$, so $\frac{\theta}{360} \times 30 = \frac{25}{2}$

$\theta = \frac{25}{2} \times \frac{360}{30} = 150°$

(b) Area = $\frac{150}{360} \times \pi \times 15^2 = \frac{375}{4}\pi$ cm^2

Answer: (a) 150° (b) $\frac{375}{4}\pi$ cm^2

2 Gradient at a point

Work out the gradient of the curve $y = 4x^3 + 4x^2 + 1$ at the point where $x = -1$.

$\frac{dy}{dx} = 12x^2 + 8x$

When $x = -1$: gradient = 4

Answer: 4

3 Median and quartiles from a list

Here are 11 numbers.
4, 33, 15, 25, 17, 18, 43, 34, 42, 24, 54
Find (a) the median (b) the interquartile range.

Ordered: 4, 15, 17, 18, 24, 25, 33, 34, 42, 43, 54
(a) Median is the 6th value = 25
(b) Lower quartile is the 3rd value = 17; upper quartile is the 9th value = 42
IQR = 42 − 17 = 25

Answer: (a) 25 (b) 25

4 Tangent perpendicular to a line

The tangent at the point P on the curve $y = -2x^2 + x + 2$ is perpendicular to the line $x - 3y = -9$.
Find the coordinates of P.

The line is $y = \frac{1}{3}x + 3$, so its gradient is $\frac{1}{3}$

The tangent has gradient -3

$\frac{dy}{dx} = -4x + 1 = -3$ so $x = 1$

$y = 1$

Answer: (1, 1)

5 Perpendicular bisector

A is the point (8, 7) and B is the point (10, 13).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{8+10}{2}, \frac{7+13}{2}) = (9, 10)$

Gradient of AB = $\frac{13-7}{10-8} = 3$

The perpendicular gradient is $-1 \div 3 = -\frac{1}{3}$

Using the midpoint: $y - 10 = -\frac{1}{3}(x - 9)$

$y = -\frac{1}{3}x + 13$

Answer: $y = -\frac{1}{3}x + 13$

6 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 40 is taken. Work out how many come from Group B.
(b) A different stratified sample has 36 from Group D. Work out the size of this sample.

Total = 240 + 160 + 160 + 240 = 800

(a) Group B: $\frac{160}{800} \times 40 = 8$

(b) Fraction sampled = $36 \div 240 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 800 = 120$

Answer: (a) 8 (b) 120

7 Growth and decay

Hana puts £10,000 into a savings account.
The account pays 6% compound interest each year.
After how many whole years will there first be more than £13,350 in the account? (Calculator)

Multiplier each year = 1.06

After 4 years: £10,000 × 1.06^4 = £12,624.77

After 5 years: £10,000 × 1.06^5 = £13,382.26

So the first time it is more than £13,350 is after 5 years

Answer: 5 years

8 Self-inverse function

$f(x) = \frac{5x+3}{x-k}$, where k is a constant and $x \neq k$.

The function f is its own inverse, so $f^{-1}(x) = f(x)$.
Find the value of k.

Let $y = \frac{5x+3}{x-k}$, so $y(x - k) = 5x + 3$

$xy - ky = 5x + 3$, so $x(y - 5) = ky + 3$

$f^{-1}(x) = \frac{kx+3}{x-5}$

For $f^{-1}(x) = f(x)$ the denominators must match: $x - 5 = x - k$

Answer: $k = 5$

9 Minimum gradient of a curve

A curve has equation $y = x^3 + 3x^2 + 13x + 8$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 3x^2 + 6x + 13$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $6x + 6 = 0$, so $x = -1$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is $6 > 0$), so this is a minimum

$\frac{dy}{dx} = 10$ when $x = -1$

Answer: minimum gradient 10 when $x = -1$

10 Transformations of graphs

The point $P(-2, -2)$ lies on the graph of $y = f(x)$.
Write down the coordinates of the image of P on the graph of

(a) $y = \frac{1}{2}f(x)$ (b) $y = f(x) + 2$

(a) $y = \frac{1}{2}f(x)$ is a vertical stretch with scale factor $\frac{1}{2}$, so the y coordinate is halved: $(-2, -1)$

(b) $y = f(x) + 2$ is a translation of 2 up: $(-2, 0)$

Answer: (a) $(-2, -1)$ (b) $(-2, 0)$