Day 290 **IGCSE 10-a-day: Model Solutions**

1 Stationary point of $y = ax + b/\sqrt{x}$

A curve has equation $y = -2x - \frac{32}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -2 + 16x^{-3/2}$

$\frac{dy}{dx} = 0$: $-16x^{-3/2} = -2$, so $x^{3/2} = 8$

$x = 8^{2/3} = 4$, and $y = -24$

$\frac{d^2y}{dx^2} = -24x^{-5/2}$

At $x = 4$, $\frac{d^2y}{dx^2} = -\frac{3}{4}$, which is negative, so a maximum

Answer: (4, −24), maximum

2 Combined transformations as matrices

$P = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation of 90° anticlockwise about the origin.

$Q = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represents a reflection in the line $y = -x$.

(a) Find the matrix that represents P followed by Q.
(b) Describe the single transformation that this matrix represents.

(a) P followed by Q is the product QP (the first transformation is on the right)

$QP = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) This matrix represents a reflection in the y-axis

Answer: (a) $\begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ **(b) Reflection in the y-axis**

3 Solving quadratics by factorising

Solve $x^2 + 11x = -28$

Rearrange so that one side is 0: $x^2 + 11x + 28 = 0$
Factorise: $(x + 7)(x + 4) = 0$
$x + 7 = 0$ or $x + 4 = 0$
$x = -7$ or $x = -4$
Answer: x = −7 or x = −4

4 Inclusion and exclusion with two sets

There are 86 families.
12 of them have a garden and 13 of them have a car.
63 of them do neither.
(a) Work out how many do both.
(b) One of the families is chosen at random. Find the probability that they only have a garden.

Number in A or B = 86 − 63 = 23
(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 12 + 13 - 23 = 2$
(b) Number who only have a garden = 12 − 2 = 10

Probability = $\frac{10}{86} = \frac{5}{43}$

Answer: (a) 2 (b) $\frac{\mathbf{5}}{\mathbf{43}}$

5 Fractions

Work out $3\frac{1}{2} \times 2\frac{1}{2} - 1\frac{7}{8}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{7}{2} \times \frac{5}{2} = \frac{35}{4}$

$\frac{35}{4} - \frac{15}{8} = \frac{55}{8}$

$= 6\frac{7}{8}$

Answer: $6\frac{7}{8}$

6 Equations with algebraic fractions

Solve $\frac{1}{x+5} + \frac{3}{x-5} = \frac{2}{3}$

Multiply every term by $(x + 5)(x - 5)$: $(x - 5) + 3(x + 5) = \frac{2}{3} \times (x + 5)(x - 5)$

Multiply by 3 to clear the fraction: $3(x - 5) + 9(x + 5) = 2(x + 5)(x - 5)$

Expand and rearrange so that one side is 0: $x^2 - 6x - 40 = 0$

Factorise: $(x - 10)(x + 4) = 0$

$x = -4$ or $x = 10$

Answer: $x = -4$ or $x = 10$

7 Minimum distance with 1/t

A particle P moves on a straight line. At time t seconds ($t > 0$), its distance from a point O is s metres, where

$s = t^2 + \frac{128}{t}$

Show that P is at its minimum distance from O when $t = 4$, and find this minimum distance.

$s = t^2 + 128t^{-1}$, so $v = \frac{ds}{dt} = 2t - 128t^{-2}$

$v = 0$: $2t = \frac{128}{t^2}$, so $t^3 = 64$ and $t = 4$

$\frac{d^2s}{dt^2} = 2 + \frac{256}{t^3}$, which is 6 when $t = 4$, so a minimum

$s = 48$

Answer: 48 m

8 Three-part ratios

Liam, Kira and Leila share some stickers.

Liam : Kira = 2 : 9

Kira : Leila = 6 : 2

(a) Work out the ratio Liam : Kira : Leila.

(b) They share 126 stickers altogether. How many stickers does Kira get?

(a) Make the Kira part the same: 2 : 9 = 4 : 18 and 6 : 2 = 18 : 6

Liam : Kira : Leila = 4 : 18 : 6 = 2 : 9 : 3

(b) Total parts = 14, so 1 part = 126 ÷ 14 = 9

Kira gets 9 × 9 = 81 stickers

Answer: (a) 2 : 9 : 3 (b) 81 stickers

9 Equation of a normal

A curve has equation $y = 3x^3 - 6x^2 + 5x$.
Find the equation of the normal to the curve at the point where x = 0.

When x = 0, y = 0 and $\frac{dy}{dx} = 5$

Gradient of the normal = $-1 \div 5 = -\frac{1}{5}$

$y = -\frac{1}{5}x$

$y = -\frac{1}{5}x$

Answer: $y = -\frac{1}{5}x$

10 Area of a segment

The circle has centre O and radius 13 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 13^2 = 206.47$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 13 \times 13 \times \sin 140° = 54.316$ cm^2

Segment = sector − triangle = 206.47 − 54.316 = 152.16
= 152 cm^2 (3 s.f.)

Answer: 152 cm^2