Day 29 **IGCSE 10-a-day: Model Solutions**

1 Bearings with the sine and cosine rules

A ship leaves a port P and travels 77 km on a bearing of 155° to Q. (Calculator)
It then travels 84 km on a bearing of 125° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 155° + 180° = 335°
Angle PQR = 150° (the difference between the bearings 335° and 125°)
$PR^2 = 77^2 + 84^2 - 2 \times 77 \times 84 \times \cos 150° = 24188$, so PR = 155.5 km

(b) $\cos(QPR) = \frac{77^2 + PR^2 - 84^2}{2 \times 77 \times PR}$, so angle QPR = 15.7°

R is anticlockwise of the direction PQ, so the bearing is 155° − 16° = 139°

Answer: (a) 155.5 km (b) 139°

2 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$.
Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix} -10 \\ 14 \end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} -10 \\ 14 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ 11 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-4} \\ \mathbf{11} \end{pmatrix}$

3 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = 2t^3 - 12t^2 + 27t - 5$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 24t + 27 = 6(t - 2)^2 + 3$

A square is never negative, so $v \geq 3 > 0$ and P is never at rest
Least value of v occurs when t = 2, giving 3

Answer: (b) 3 m/s when t = 2 s

4 Simultaneous equations

Solve the simultaneous equations
$-5x - y = 24$
$3x + 4y = 6$

To match the y terms: (1) × 4 gives $-20x - 4y = 96$
Add the equations to eliminate y: $-17x = 102$
$x = -6$
Substitute $x = -6$ into (1): $-y + 30 = 24$
$-y = -6$, so $y = 6$

Answer: x = −6, y = 6

5 Percentage change and proportion

The area of a circle is directly proportional to the square of the radius.
The radius is increased by 30%.
Work out the percentage change in the area of a circle.

The radius is multiplied by 1.3
The multiplier for the area of a circle is $1.3^2 = 1.69$
1.69 is an increase of 69%

Answer: 69% increase

6 Stationary points of a quartic

A curve has equation $y = 2x^4 - 16x^2 + 3$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 32x = 8x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 2$ or $x = -2$

$y = 3$ when $x = 0$, and $y = -29$ when $x = \pm 2$

$\frac{d^2y}{dx^2} = 24x^2 - 32$: it is -32 at $x = 0$ (maximum) and 64 at $x = \pm 2$ (minimum)

Answer: (0, 3) maximum; (2, −29) and (−2, −29) minima

7 Relative speed

Zane and Amir live 320 km apart.
At 10:00 they set off towards each other on the same road.
Zane drives at 85 km/h and Amir drives at 75 km/h.
(a) At what time do they meet?
(b) How far from Zane's home do they meet?

Closing speed = 85 + 75 = 160 km/h
(a) Time = 320 ÷ 160 = 2 hours = 120 minutes, so they meet at 12:00
(b) Distance = 85 × 2 = 170 km
Answer: (a) 12:00 (b) 170 km

8 Normal meets the x-axis

The normal to the curve $y = x^3 - 2x + 2$ at the point where $x = -1$ meets the x-axis at N.
Find the coordinates of N.

$y = 3$ and gradient of curve = 1
Gradient of normal = −1
$y - 3 = -(x + 1)$
Put $y = 0$: $-3 = -(x + 1)$
$x = 2$
Answer: (2, 0)

9 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.
Over the two years the population increases by 125%.
Work out the value of x.

Yearly multiplier = $1 + \frac{x}{100}$

Two years: $(1 + \frac{x}{100})^2 = 2.25$

Take the square root: $1 + \frac{x}{100} = 1.5$

$\frac{x}{100} = 0.5$, so $x = 50$

Answer: x = 50

10 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 12 cm, BC = 7 cm, CD = 12 cm,
angle ABC = 54° and angle ACD = 44°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 12^2 + 7^2 - 2 \times 12 \times 7 \times \cos 54°$ so AC = 9.7084 cm
Area ABC = ½ × 12 × 7 × sin 54° = 33.98
Area ACD = ½ × 9.7084 × 12 × sin 44° = 40.46
Total area = 33.98 + 40.46 = 74.4
Answer: 74.4 cm^2