Day 289 **IGCSE 10-a-day: Model Solutions**

1 Point with a given gradient (quadratic)

A curve has equation $y = -x^2 + 5x - 5$.
The gradient at the point P on the curve is 11.
Find the coordinates of P.

$\frac{dy}{dx} = -2x + 5$
$-2x + 5 = 11$, so $-2x = 6$
$x = -3$
$y = -29$
Answer: $(-3, -29)$

2 Mean of a changing set

The mean of 10 numbers is 173.
One number is removed and the mean of the remaining 9 numbers is 187.
Work out the number that was removed.

Total of the 10 numbers = $10 \times 173 = 1730$
Total of the 9 remaining = $9 \times 187 = 1683$
Number removed = $1730 - 1683 = 47$
Answer: 47

3 Fractional power functions

$f(x) = -4x^{5/2} - x + 6x^{-3/2}$
Find $f'(x)$ and work out the value of $f'(9)$.

$-4x^{5/2} \to -10x^{3/2}$, $-x \to -1$, $6x^{-3/2} \to -9x^{-5/2}$
$f'(x) = -10x^{3/2} - 1 - 9x^{-5/2}$
$f'(9)$: $\sqrt{9} = 3$, so $f'(9) = -\frac{7318}{27}$

Answer: $f'(x) = -10x^{3/2} - 1 - 9x^{-5/2}$, $f'(9) = -\frac{7318}{27}$

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 51°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 51 = 102°$
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = $360 - 90 - 90 - 102 = 78°$
Answer: (a) 102° (b) 78°

5 Surds

Given that $\sqrt{48} + \sqrt{27} = \sqrt{n}$,
find the value of n.

$\sqrt{48} = 4\sqrt{3}$ and $\sqrt{27} = 3\sqrt{3}$
$4\sqrt{3} + 3\sqrt{3} = 7\sqrt{3}$
$7\sqrt{3} = \sqrt{147}$, so $n = 147$
Answer: $n = 147$

6 Vector between two points

The position vectors of A and B are $\begin{pmatrix}-6\\1\end{pmatrix}$ and $\begin{pmatrix}7\\2\end{pmatrix}$.

(a) Write down the vector **AB**.

(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix}7\\2\end{pmatrix} - \begin{pmatrix}-6\\1\end{pmatrix} = \begin{pmatrix}13\\1\end{pmatrix}$

(b) $|\mathbf{AB}| = \sqrt{(13)^2 + (1)^2} = \sqrt{170}$

Answer: (a) $\begin{pmatrix}\mathbf{13}\\\mathbf{1}\end{pmatrix}$ **(b)** $\sqrt{\mathbf{170}}$

7 Rearranging when the subject appears twice

Make t the subject of $T(t + 2) = t - T$

Expand the bracket: $Tt + 2T = t - T$

Collect the t terms: $Tt - t = -2T - T$

Factorise: $t(T - 1) = -3T$

$t = \frac{3T}{1 - T}$

Answer: $\mathbf{t = \frac{3T}{1 - T}}$

8 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{6}{11}$.

(a) Find the exact value of cos x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{6}{11})^2 = \frac{85}{121}$

x is obtuse, so cos x is negative: $\cos x = -\frac{\sqrt{85}}{11}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{6}{11} \div (-\frac{\sqrt{85}}{11}) = -\frac{6\sqrt{85}}{85}$

Answer: (a) $-\mathbf{\frac{\sqrt{85}}{11}}$ **(b)** $-\mathbf{\frac{6\sqrt{85}}{85}}$

9 Two picks without replacement

A bag contains 7 blue and 7 green counters.

Two counters are taken at random without replacement.

Work out the probability that the two counters are the same colour.

P(same colour) = sum over each colour of P(both that colour)

$\frac{7}{14} \times \frac{6}{13} + \frac{7}{14} \times \frac{6}{13}$

$= \frac{6}{13}$

Answer: $\mathbf{\frac{6}{13}}$

10 Tangent where the curve crosses the y-axis

The curve $y = 3x^2 - x - 2$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = -2$

$\frac{dy}{dx} = 6x - 1$, so the gradient at A is -1

Tangent: $y = -x - 2$

At B, $y = 0$: $-x - 2 = 0$, so $x = -2$

Answer: $(-2, 0)$