Day 288 **IGCSE 10-a-day: Model Solutions**

1 Solve f inverse equals g

$f(x) = \frac{x+5}{4}$ and $g(x) = x^2 + 3x - 17$.

Solve $f^{-1}(x) = g(x)$.

$f^{-1}(x) = 4x - 5$

$4x - 5 = x^2 + 3x - 17$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = -3$ or $x = 4$

Answer: $x = -3$ or $x = 4$

2 Comparing costs

Zip Taxis has a fixed charge of £1.50 plus £5 per mile.
Go Cabs has a fixed charge of £37.50 plus £2 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 15 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $1.50 + 5n = 37.50 + 2n$
$3n = 36$, so $n = 12$
(b) Zip Taxis: £1.50 + £5 × 15 = £76.50
Go Cabs: £37.50 + £2 × 15 = £67.50
Go Cabs is cheaper by £9
Answer: (a) 12 (b) Go Cabs, by £9

3 Standard form: multiplying and dividing

Work out $(2 \times 10^{-3}) \times (1.2 \times 10^{-1})$.
Give your answer in standard form.

Multiply the numbers: $2 \times 1.2 = 2.4$
Add the powers: $10^{-3} \times 10^{-1} = 10^{-4}$
$2.4 \times 10^{-4} = 2.4 \times 10^{-4}$
Answer: 2.4×10^{-4}

4 Increasing and decreasing with powers

$f(x) = 2x + \frac{18}{x}$ for $x > 0$

(a) Find the range of values of x for which f is a decreasing function.
(b) Write down the least value of f(x).

$f'(x) = 2 - 18x^{-2} = 2 - \frac{18}{x^2}$

f is decreasing when $f'(x) < 0$: $2 < \frac{18}{x^2}$, so $x^2 < 9$

With $x > 0$ this gives $0 < x < 3$
f decreases then increases, so the least value is at $x = 3$: $f(3) = 12$
Answer: (a) $0 < x < 3$ (b) 12

5 Area of a segment

The circle has centre O and radius 9 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 9^2 = 98.960$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 9 \times 9 \times \sin 140° = 26.033$ cm^2

Segment = sector − triangle = 98.960 − 26.033 = 72.927
= 72.9 cm^2 (3 s.f.)
Answer: 72.9 cm^2

6 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 55 − 35 = 20
IQR of B = 75 − 65 = 10
(b) Class B has the higher median (70 compared with 50), so on average Class B scored higher
Class B has the smaller interquartile range (10 compared with 20), so its marks are more consistent
Answer: (a) A: 20, B: 10 (b) B has the higher median, B has the smaller IQR

7 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, 4) lies on the curve and the gradient of the curve at this point is −1.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$
Point on curve: a + b = 4. Gradient: 2a − b = −1
Solving simultaneously gives a = 1 and b = 3
Answer: a = 1, b = 3

8 Laws of indices

Write $\frac{8^n \times 4^{n-2}}{2^n}$ in the form 2^{an+c}

where a and c are integers.

$8 = 2^3$, $4 = 2^2$, $2 = 2^1$
$8^n = 2^{3(n)}$, $4^{n-2} = 2^{2(n-2)}$, $2^n = 2^n$
Exponent: 3(n) + 2(n − 2) − n = 4n − 4
Answer: a = 4, c = −4

9 Normal to y = kx^2: show that

P is the point $(a, \frac{7}{2}a^2)$ on the curve $y = \frac{7}{2}x^2$, where $a \neq 0$.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{7}{2}a^2 + \frac{1}{7}$.

$\frac{dy}{dx} = 7x$, so the gradient of the tangent at P is 7a

Normal gradient = $-\frac{1}{7a}$

Normal: $y - \frac{7}{2}a^2 = -\frac{1}{7a}(x - a)$

Put x = 0: $y = \frac{7}{2}a^2 + \frac{a}{7a} = \frac{7}{2}a^2 + \frac{1}{7}$

Answer: shown: $y = \frac{7}{2}a^2 + \frac{1}{7}$

10 Equations with algebraic fractions

Solve $\frac{5}{x+1} + \frac{2}{x-2} = \frac{1}{2}$

Multiply every term by (x + 1)(x − 2): $5(x - 2) + 2(x + 1) = \frac{1}{2} \times (x + 1)(x - 2)$
Multiply by 2 to clear the fraction: 10(x − 2) + 4(x + 1) = (x + 1)(x − 2)
Expand and rearrange so that one side is 0: $x^2 - 15x + 14 = 0$
Factorise: (x − 14)(x − 1) = 0
x = 1 or x = 14
Answer: x = 1 or x = 14