Day 287 **IGCSE 10-a-day: Model Solutions**

1 Transformed curve from a diagram

The diagram shows the graph of y = f(x) with a minimum point at (0, 4).
The dashed curve C is the image of y = f(x) after a translation.
The minimum point of C is (−4, 1).
Write down the equation of C in terms of f.

The minimum point has moved from (0, 4) to (−4, 1)

Translation by $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

x moves left by 4: f(x + 4). y moves down by 3: − 3

Answer: y = f(x + 4) − 3

2 Expand then differentiate

y = x(x + 4)(x − 1)

Find $\frac{dy}{dx}$.

Expand: $y = x^3 + 3x^2 - 4x$

$x^3 \rightarrow 3x^2$, $3x^2 \rightarrow 6x$, $-4x \rightarrow -4$

$\frac{dy}{dx} = 3x^2 + 6x - 4$

Answer: $\mathbf{\frac{dy}{dx} = 3x^2 + 6x - 4}$

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (4x − 20)°, angle BCD = (5x − 7)° and angle ABC = (3x − 15)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(4x − 20) + (5x − 7) = 180
9x − 27 = 180, so 9x = 207
x = 23
(b) Angle ABC = 3 × 23 − 15 = 54°
Angle ADC = 180 − 54 = 126°
Answer: (a) x = 23 (b) 126°

4 Gradient with fractional or negative powers

The equation of a curve is $y = -2x^{3/2} - 4x + 2\sqrt{x}$.
Work out the gradient of the curve at the point where x = 16.

$\frac{dy}{dx} = -3\sqrt{x} - 4 + x^{-1/2}$

When x = 16: gradient = $-\frac{63}{4}$

Answer: $\mathbf{-\frac{63}{4}}$

5 Upper and lower bounds

a = 0.92 and b = 0.27,
both correct to 2 decimal places.
Work out the upper bound of a × b.

Each value is rounded, so it can be up to 0.005 above or below
0.915 ≤ a < 0.925; 0.265 ≤ b < 0.275
Upper bound = 0.925 × 0.275 = 0.254375
Answer: 0.254375

6 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles E, F and G
The shaded region is (E ∪ G) ∩ F
Answer: (E ∪ G) ∩ F

7 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 38°.
B is a point on the same line, 80 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 25°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 38° = \frac{h}{x}$, so $x = h \div \tan 38°$

From B: $\tan 25° = \frac{h}{x + 80}$, so $x + 80 = h \div \tan 25°$

Subtract: $80 = h \div \tan 25° - h \div \tan 38° = h(2.1445 - 1.2799)$
$h = 80 \div 0.86457 = 92.5$ m (1 d.p.)
Answer: 92.5 m

8 Increasing range for y = ax + b/x

$y = x + \frac{9}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 1 - \frac{9}{x^2}$

$\frac{dy}{dx} > 0$: $1 > \frac{9}{x^2}$

$x^2 > 9$ (x^2 is positive, so multiply through)
$x < -3$ or $x > 3$
Answer: $x < -3$ or $x > 3$

9 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 10x + 24}{x^2 + 2x} \times \frac{x^2 - 3x}{x - 4}$

Factorise: $\frac{(x - 4)(x - 6)}{x(x + 2)} \times \frac{x(x - 3)}{(x - 4)}$
Cancel $(x - 4)$ and x
$= \frac{(x - 6)(x - 3)}{(x + 2)}$
Answer: $\frac{(x - 6)(x - 3)}{(x + 2)}$

10 Finding the rate of interest

Liam invested £16,000 in a savings account for 3 years.
Compound interest was paid at a fixed rate per year.
At the end of the 3 years the account held £18,522.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $16000 \times m^3 = 18522.00$
$m^3 = 18522.00 \div 16000 = 1.157625$
$m = \sqrt[3]{1.157625} = 1.05$
Rate $= (1.05 - 1) \times 100 = 5\%$
Answer: 5% per year