Day 286 **IGCSE 10-a-day: Model Solutions**

1 Percentage change

The price of a phone is decreased by 5%.
The new price is then decreased by 40%.
Work out the overall percentage change in the price.

Multipliers: 0.95 and 0.6
Overall multiplier = 0.95 × 0.6 = 0.57
0.57 − 1 = −0.43, so an overall decrease of 43%
Answer: 43% decrease

2 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 2 cm, DB = 2 cm and the area of triangle ADE is 18 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 2 + 2 = 4 cm
Length scale factor = 4 ÷ 2 = 2
Area scale factor = $(2)^2 = 4$
Area of triangle ABC = 18 × 4 = 72 cm^2
Trapezium = 72 − 18 = 54 cm^2
Answer: 54 cm^2

3 Increasing or decreasing at a point

$y = 2x^3 + 12x^2 + 18x$
At the point where x = 1, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 + 24x + 18$

When x = 1: $\frac{dy}{dx} = 48$

48 is positive, so the curve is increasing
Answer: increasing (gradient 48)

4 Two picks without replacement

A bag contains 5 blue, 2 green and 6 black cubes.
Two cubes are taken at random without replacement.
Work out the probability that one is blue and the other is green.

Two orders: blue then green, or green then blue

$\frac{5}{13} \times \frac{2}{12} + \frac{2}{13} \times \frac{5}{12}$

$= \frac{5}{39}$

Answer: $\frac{5}{39}$

5 Trigonometric equations

Solve $2\cos x° - 1 = -1$ for $0 \leq x \leq 360$

Rearrange: $2\cos x° = 0$, so $\cos x° = 0$
From the graph of $y = \cos x°$, $\cos x° = 0$ when x = 90 or 270
Answer: x = 90, 270

6 Weighted percentages

The ratio of men to women in a school is 4 : 1.
20% of the men and 55% of the women own a car.
What percentage of all of these students own a car?

Imagine 100 students in total: 80 men and 20 women
20% of 80 = 16 and 55% of 20 = 11
Together: 27 out of 100
27 ÷ 100 × 100 = 27%
Answer: 27%

7 Composite with an inverse function

$f(x) = 5x - 3$ and $g(x) = 3x^2 + 4x + 4$.
(a) Find $f^{-1}g(x)$.
(b) Work out $f^{-1}g(3)$.

(a) $f^{-1}(x) = \frac{x + 3}{5}$

$f^{-1}g(x) = \frac{3x^2 + 4x + 4 + 3}{5} = \frac{3x^2 + 4x + 7}{5}$

(b) $g(3) = 43$, so $f^{-1}g(3) = \frac{43 + 3}{5}$

Answer: (a) $\frac{3x^2 + 4x + 7}{5}$ **(b)** $\frac{46}{5}$

8 No stationary points

A curve has equation $y = 2x^3 + x + 6$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 + 1 = 6x^2 + 1$

A squared term is never negative, so $\frac{dy}{dx} \geq 1 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 1 (when x = 0)
Answer: (b) 1

9 Mixtures and percentage profit

A shop mixes two types of coffee.
Type A costs £14.00 per kg and type B costs £5.00 per kg.
They are mixed in the ratio A : B = 3 : 2.
The mixture is sold for £11.96 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 3 × £14.00 + 2 × £5.00 = £52.00
Cost per kg = £52.00 ÷ 5 = £10.40
Profit per kg = £11.96 − £10.40 = £1.56
Percentage profit = 1.56 ÷ 10.40 × 100 = 15%
Answer: 15%

10 Maximum revenue

A company sells x thousand items each month. The price of each item is £12 − x.
The monthly revenue is £R thousand.

(a) Show that $R = 12x - x^2$.

(b) Find the maximum monthly revenue, showing that it is a maximum.

$R = x \times (12 - x) = 12x - x^2$

$\frac{dR}{dx} = -2x + 12 = 0$ gives $x = 6$

$\frac{d^2R}{dx^2} = -2 < 0$, so a maximum

$R = 12 \times 6 - 1 \times 6^2 = 36$

Answer: £36 thousand