Day 285 **IGCSE 10-a-day: Model Solutions**

1 Column vectors

$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.

Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix} 2 + 1k \\ -1 + 1k \end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{2 + 1k}{1} = \frac{-1 + 1k}{-2}$

$-2(2 + 1k) = 1(-1 + 1k)$

$k \times (-3) = 3$

$k = -1$

Answer: $k = -1$

2 Negative powers

Differentiate $x^3 + x + \frac{8}{x^3}$ with respect to x.

Write negative powers as indices: $y = x^3 + x + 8x^{-3}$

$x^3 \to 3x^2$, $x \to 1$, $8x^{-3} \to -24x^{-4}$

$\frac{dy}{dx} = 3x^2 + 1 - 24x^{-4}$

Answer: $\frac{dy}{dx} = 3x^2 + 1 - \frac{24}{x^4}$

3 Inequalities

(a) Solve $4 - 6x < -14$

(b) n is an integer. Write down all the values of n that satisfy

$-20 \leq 4n \leq 8$

(a) Subtract 4: $-6x < -18$

Divide by -6 and reverse the inequality sign: $x > 3$

(b) Subtract 0: $-20 \leq 4n \leq 8$

Divide by 4: $-5 \leq n \leq 2$

Integers: $-5, -4, -3, -2, -1, 0, 1, 2$

Answer: (a) $x > 3$ (b) $-5, -4, -3, -2, -1, 0, 1, 2$

4 Bounds in calculations

A runner covers 380 m, correct to the nearest 10 m, in 6.7 seconds, correct to 1 decimal place.

Work out the upper bound and the lower bound of the runner's average speed, in m/s.

Give each answer correct to 3 significant figures. (Calculator)

Distance: $375 \leq d < 385$. Time: $6.65 \leq t < 6.75$

UB speed = UB distance ÷ LB time = 385 ÷ 6.65 = 57.9

LB speed = LB distance ÷ UB time = 375 ÷ 6.75 = 55.6

Answer: UB = 57.9 m/s, LB = 55.6 m/s

5 Income tax

In one country, income tax is worked out like this:
The first £12,000 of income is not taxed.
The next £25,000 is taxed at 20%.
All income above that is taxed at 40%.
Ivan earns £75,500 in a year.
Work out how much income tax Ivan pays.

Taxed at 20%: £25,000
20% of £25,000 = £5,000
Taxed at 40%: £75,500 − £12,000 − £25,000 = £38,500
40% of £38,500 = £15,400
Total tax = £5,000 + £15,400 = £20,400
Answer: £20,400

6 Largest domain and range

$f(x) = \sqrt{-x + 6}$.
(a) State the largest possible domain of f.
(b) State the range of f for this domain.

(a) Need $-x + 6 \geq 0$, so $x \leq 6$
Domain: $x \leq 6$
(b) A square root is never negative
Answer: (a) $x \leq 6$ (b) $f(x) \geq 0$

7 Open box from a square sheet

Squares of side x cm are cut from the corners of a 12 cm square cardboard sheet, which is folded into an open box of volume V cm^3.
(a) Show that $V = 4x^3 - 48x^2 + 144x$.
(b) Find x for the maximum volume, and this volume.

Base is a square of side $12 - 2x$, so $V = x(12 - 2x)^2 = x(144 - 48x + 4x^2) = 4x^3 - 48x^2 + 144x$
$\frac{dV}{dx} = 12x^2 - 96x + 144 = 0$, so $x^2 - 8x + 12 = 0$
$12(x - 2)(x - 6) = 0$, so $x = 2$ or $x = 6$
$x = 6$ is rejected (6 is more than half of 12). $\frac{d^2V}{dx^2} = 24x - 96$, which is -48 when $x = 2$, so a maximum
$V = 128$
Answer: $x = 2$, $V = 128\ cm^3$

8 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 3, 20, 3, 8
Mid-points: 2.5, 10, 22.5, 40
$\Sigma fm = 3\times2.5 + 20\times10 + 3\times22.5 + 8\times40 = 595$, $\Sigma f = 34$
Mean = 595 ÷ 34 = 17.5
Answer: 17.5

9 Where two tangents meet

The curve C has equation $y = 2x^2 - 3x - 6$.
The points A and B on C have x coordinates -3 and 0.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 4x - 3$. At A: gradient -15, A = $(-3, 21)$
Tangent at A: $y = -15x - 24$. At B: gradient -3, B = $(0, -6)$
Tangent at B: $y = -3x - 6$
Equate: $-15x - 24 = -3x - 6$, so $x = -\frac{3}{2}$ and $y = -\frac{3}{2}$

Answer: T $\left(-\frac{3}{2}, -\frac{3}{2}\right)$

10 Cone made from a sector

A sector of a circle has radius 20 cm and angle 72°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{72}{360} \times 2\pi \times 20 = 8\pi$ cm
This is the circumference of the base: $2\pi r = 8\pi$, so r = 4 cm
(b) Slant height = 20 cm, so height$^2 = 20^2 - 4^2 = 384$, height = $8\sqrt{6}$ cm
Volume = $\frac{1}{3} \times \pi \times 4^2 \times 8\sqrt{6} = \frac{128\sqrt{6}}{3}\pi$ cm^3

Answer: (a) 4 cm (b) $\frac{128\sqrt{6}}{3}\pi$ cm^3