Day 284 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $m^2 - 3nk$
when $m = 3$, $n = 1$ and $k = -3$.

Substitute the values: $3^2 - 3 \times 1 \times (-3)$
$= 9 - (-9)$
$= 18$
Answer: 18

2 Differentiating polynomials

Differentiate $-2x^5 + 6x^4 - 7x$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$
$-2x^5 \to -10x^4$, $6x^4 \to 24x^3$, $-7x \to -7$
$\frac{dy}{dx} = -10x^4 + 24x^3 - 7$
Answer: $\mathbf{\frac{dy}{dx} = -10x^4 + 24x^3 - 7}$

3 Inequalities

(a) Solve $8 - 2x < 8$
(b) n is an integer. Write down all the values of n that satisfy
$-3 \leq 2n + 5 \leq 3$

(a) Subtract 8: $-2x < 0$
Divide by -2 and reverse the inequality sign: $x > 0$
(b) Subtract 5: $-8 \leq 2n \leq -2$
Divide by 2: $-4 \leq n \leq -1$
Integers: $-4, -3, -2, -1$
Answer: (a) $x > 0$ (b) $-4, -3, -2, -1$

4 Fractional indices

Given that $x > 0$, solve $x^{\frac{2}{3}} = 4$

Raise both sides to the power $\frac{3}{2}$

$x = (4)^{\frac{3}{2}}$
$x = 8$
Answer: $x = 8$

5 Tangent to a curve with negative or fractional powers

A curve has equation $y = -2x^2 - \frac{4}{x}$.

Find the equation of the tangent to the curve at the point where $x = 4$.

$\frac{dy}{dx} = -4x + 4x^{-2}$

At $x = 4$: $y = -33$ and gradient $= -\frac{63}{4}$

$y + 33 = -\frac{63}{4}(x - 4)$

$y = -\frac{63}{4}x + 30$

Answer: $\mathbf{y = -\frac{63}{4}x + 30}$

6 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and CB = $\frac{3}{4}$OA.

$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.

(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

(b) AM : MB = 1 : 1. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = \frac{3}{4}\mathbf{a}$

$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \frac{3}{4}\mathbf{a} + \mathbf{c}$

$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = -\frac{1}{4}\mathbf{a} + \mathbf{c}$

(b) $\mathbf{OM} = \mathbf{OA} + \frac{1}{2}\mathbf{AB} = \mathbf{a} + \frac{1}{2}(-\frac{1}{4}\mathbf{a} + \mathbf{c})$

$= \frac{7}{8}\mathbf{a} + \frac{1}{2}\mathbf{c}$

Answer: (a) $-\frac{1}{4}\mathbf{a} + \mathbf{c}$ (b) $\frac{7}{8}\mathbf{a} + \frac{1}{2}\mathbf{c}$

7 Nature from the gradient either side

$y = x^3 - 3x + 6$

(a) Show that there is a stationary point at x = 1.

(b) By finding the gradient when x = $\frac{1}{2}$ and when x = $\frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 - 3$, so when x = 1: $\frac{dy}{dx} = 0$

x = $\frac{1}{2}$: gradient = $-\frac{9}{4}$ (negative)

x = $\frac{3}{2}$: gradient = $\frac{15}{4}$ (positive)

The gradient goes from negative to 0 to positive, so (1, 4) is a minimum

Answer: (b) minimum

8 Combined transformations

The point P(−1, 6) is rotated 180° about the origin.

The image is then reflected in the y-axis.

(a) Write down the coordinates of the final image of P.

(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (1, −6)

After the second transformation: P'' = (−1, −6)

(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, −y) → (x, −y)

The single mapping is (x, y) → (x, −y)

This is a reflection in the x-axis

Answer: (a) (−1, −6) (b) Reflection in the x-axis

9 Standard form: adding and subtracting

Work out $9.8 \times 10^0 - 1.9 \times 10^{-1}$.

Give your answer in standard form.

Write both numbers with the same power of 10, 10^{-1}:

$98 \times 10^{-1} - 1.9 \times 10^{-1}$

$= (98 - 1.9) \times 10^{-1} = 96.1 \times 10^{-1}$

$= 9.61 \times 10^0$

Answer: 9.61×10^0

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10 Ratio and expressions

5x = 3y

Work out the ratio (4x − 1y) : (3x + 3y)

Give your answer in its simplest form.

5x = 3y gives x : y = 3 : 5, so let x = 3t and y = 5t

4(3t) − 1(5t) = 7t

3(3t) + 3(5t) = 24t

7t : 24t = 7 : 24

Answer: 7 : 24