Day 283 **IGCSE 10-a-day: Model Solutions**

1 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = t^3 - 15t^2 + 63t + 5$ $(t \geq 0)$

Find the distance between the two positions where P is at rest.

$v = \frac{ds}{dt} = 3t^2 - 30t + 63$

P is at rest when $v = 0$: $3t^2 - 30t + 63 = 0$

$3(t - 3)(t - 7) = 0$, so $t = 3$ or $t = 7$

$s(3) = 86$ and $s(7) = 54$

Distance $= |54 - 86| = 32$

Answer: 32 m

2 Ordering numbers

Write these numbers in order of size, starting with the largest.

$\frac{7}{16}$, 0.84, $\frac{3}{11}$, 0.86

Convert each to a decimal: $\frac{7}{16} = 0.4375$, $0.84 = 0.84$, $\frac{3}{11} = 0.273$, $0.86 = 0.86$

In order: 0.86, 0.84, $\frac{7}{16}$, $\frac{3}{11}$

Answer: 0.86, 0.84, $\frac{7}{16}$, $\frac{3}{11}$

3 Show that lines are parallel

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.

M is on OA and N is on OB, with OM : MA = ON : NB = 3 : 2.

(a) Show that MN is parallel to AB.

(b) Write down the ratio MN : AB.

(a) $\mathbf{OM} = \frac{3}{5}\mathbf{a}$ and $\mathbf{ON} = \frac{3}{5}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\frac{3}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$

$\mathbf{b} - \mathbf{a} = \mathbf{AB}$, so $\mathbf{MN} = \frac{3}{5}\mathbf{AB}$: a multiple of $\mathbf{AB}$, so MN is parallel to AB

(b) MN : AB = 3 : 5

Answer: (b) 3 : 5

4 Exchange rates

Beth changes £400 into euros at £1 = €1.35.

Beth spends €170 and then changes the rest back into pounds at £1 = €1.6.

How many pounds does Beth get back?

£400 × 1.35 = €540

540 − 170 = €370

370 ÷ 1.6 = £231.25

Answer: £231.25

5 Volume and surface area of composite solids

A solid cylinder of radius 9 cm and height 13 cm has a hemispherical hollow of the same radius cut out of the top.

Work out the volume of the solid that is left. Give your answer in terms of π.

Cylinder: $\pi \times 9^2 \times 13 = 1053\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 9^3 = 486\pi$

Volume left $= 1053\pi - 486\pi = 567\pi$ cm^3

Answer: 567π cm^3

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6 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (2, 10) lies on the curve and the gradient of the curve at this point is 7.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$

Point on curve: $4a + \frac{b}{2} = 10$. Gradient: $4a - \frac{b}{4} = 7$

Solving simultaneously gives a = 2 and b = 4

Answer: a = 2, b = 4

7 Quadratic inequalities

Solve $3x^2 - 12x - 36 \geq 0$

Divide by 3: $x^2 - 4x - 12 \geq 0$

Solve $x^2 - 4x - 12 = 0$: $(x + 2)(x - 6) = 0$, so $x = -2$ or $x = 6$

The graph is a U shape crossing the x-axis at -2 and 6; it is above the axis outside the roots

$x \leq -2$ or $x \geq 6$

Answer: $x \leq -2$ or $x \geq 6$

8 Tree diagrams and reverse conditional probability

The probability that it rains is $\frac{1}{2}$.

If it rains, the probability that Sam is late is $\frac{1}{3}$. If it does not rain, the probability is $\frac{5}{6}$.

(a) Work out the probability that Sam is late.
(b) Given that Sam is late, work out the probability that it rained.

(a) P(rain and late) $= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$; P(no rain and late) $= \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

P(late) $= \frac{1}{6} + \frac{5}{12} = \frac{7}{12}$

(b) P(rain | late) $= \frac{\frac{1}{6}}{\frac{7}{12}} = \frac{2}{7}$

Answer: (a) $\frac{7}{12}$ (b) $\frac{2}{7}$

9 Laws of indices

Write $\frac{16^{n+1} \times 4^{n+1}}{2^n}$ in the form 2^{an+c}

where a and c are integers.

$16 = 2^4$, $4 = 2^2$, $2 = 2^1$

$16^{n+1} = 2^{4(n+1)}$, $4^{n+1} = 2^{2(n+1)}$, $2^n = 2^n$

Exponent: $4(n + 1) + 2(n + 1) - n = 5n + 6$

Answer: a = 5, c = 6

10 Tangent through a given point: find the constant

A curve has equation $y = -x^2 + 2x + k$, where k is a constant.
The tangent to the curve at the point where $x = -1$ passes through $(-3, -13)$.
Find the value of k.

$\frac{dy}{dx} = -2x + 2$, so at $x = -1$ the gradient is 4

The point of tangency is $(-1, -3 + k)$

Gradient between this point and $(-3, -13)$: $\frac{-13 - (-3 + k)}{-3 - (-1)} = 4$

$-13 - (-3 + k) = -8$

$k = -2$

Answer: $k = -2$