Day 282 **IGCSE 10-a-day: Model Solutions**

1 Composite functions

$f(x) = 4x + 1$ and $g(x) = x^2 + 4$

(a) Find $gf(x)$. Give your answer in the form $ax^2 + bx + c$.

(b) Solve $gf(x) = 229$

(a) $gf(x) = g(4x + 1) = (4x + 1)^2 + 4$

$= 16x^2 + 8x + 5$

(b) $(4x + 1)^2 = 225$, so $4x + 1 = \pm 15$

$4x + 1 = 15$ or $4x + 1 = -15$

$x = -4$ or $x = \frac{7}{2}$

Answer: (a) $16x^2 + 8x + 5$ (b) $x = -4$ or $x = \frac{7}{2}$

2 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where

$s = 8t^{\frac{3}{2}} + 2t^2$ $(t \geq 0)$

Find the velocity and the acceleration of the particle when $t = 9$.

$v = \frac{ds}{dt} = 4t + 12t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = 4 + 6t^{-\frac{1}{2}}$

When $t = 9$: $v = 72$ and $a = 6$

Answer: $v = 72$ m/s, $a = 6$ m/s^2

3 Distance-time graphs

The graph shows Isla's cycle ride to a café and back.

(a) Work out Isla's speed on the way back.

(b) Work out Isla's average speed for the whole ride, including the stop.

(a) Return: 12 km in 1.5 hours, so speed = 12 ÷ 1.5 = 8 km/h

(b) Total distance = 2 × 12 = 24 km, total time = 2.5 hours

Average speed = 24 ÷ 2.5 = 9.6 km/h

Answer: (a) 8 km/h (b) 9.6 km/h

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 70°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angles in quadrilateral OAPB: 360 − 90 − 90 − 70 = 110°

(b) The angle at the circumference is half the angle at the centre: 110 ÷ 2 = 55°

Answer: (a) 110° (b) 55°

5 Expanding brackets with surds

Expand and simplify $(6 + 3\sqrt{10})(5 - 3\sqrt{10})$

Give your answer in the form $p + q\sqrt{10}$

$(6 + 3\sqrt{10})(5 - 3\sqrt{10}) = 30 - 18\sqrt{10} + 15\sqrt{10} - 90$

$= -60 - 3\sqrt{10}$

Answer: $-60 - 3\sqrt{10}$

6 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the heights, in cm, of 40 plants.
(a) Find the median. (b) Find the interquartile range.
(c) How many plants are taller than 100?

(a) Median at cumulative frequency 20: reading across gives 100
(b) Lower quartile at 10 gives 80, upper quartile at 30 gives 120
IQR = 120 − 80 = 40
(c) At 100 the cumulative frequency is 20, so 40 − 20 = 20
Answer: (a) 100 (b) 40 (c) 20

7 Tangent perpendicular to a line

The tangent at the point P on the curve $y = -x^2 + 4x + 5$ is perpendicular to the line $x + 8y = 5$.
Find the coordinates of P.

The line is $y = -\frac{1}{8}x + \frac{5}{8}$, so its gradient is $-\frac{1}{8}$
The tangent has gradient 8
$\frac{dy}{dx} = -2x + 4 = 8$ so $x = -2$
$y = -7$
Answer: $(-2, -7)$

8 Percentage change in area and volume

The radius of a circle is increased by 60%.
Work out the percentage change in the area of the circle.

Radius multiplier = 1.6
Area multiplier = $1.6^2 = 2.56$
2.56 − 1 = 1.56, so a 156% increase
Answer: 156% increase

9 Angles of elevation from two points

Points A and B are on level ground, 39 m apart, with B between A and the foot of a tower.
The angle of elevation of the top of the tower is 34° from A and 52° from B.
Work out the height of the tower. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 52° − 34° = 18° (exterior angle of the triangle)
Sine rule in triangle ABT: $\frac{AT}{\sin 128°} = \frac{39}{\sin 18°}$
$AT = \frac{39 \times \sin 52°}{\sin 18°} = 99.452$
Height = AT × sin 34° = 55.6 m
Answer: 55.6 m

10 Increasing for all x

$f(x) = x^3 + kx^2 + 3x + 2$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 3$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 3 < 0$, so $4k^2 < 36$
$k^2 < 9$, so $-3 < k < 3$
Answer: $-3 < k < 3$