Day 281 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = 6x^3 + 2x - \frac{7}{x^2}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = 6x^3 + 2x - 7x^{-2}$

$6x^3 \rightarrow 18x^2$, $2x \rightarrow 2$, $-7x^{-2} \rightarrow 14x^{-3}$

$\frac{dy}{dx} = 18x^2 + 2 + 14x^{-3}$

Answer: $\mathbf{\frac{dy}{dx} = 18x^2 + 2 + \frac{14}{x^3}}$

2 Comparing costs

Hire Shop A has a delivery charge of £4.50 plus £5 per day.
Hire Shop B has a delivery charge of £16.50 plus £4 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 16 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when $4.50 + 5n = 16.50 + 4n$
$n = 12$
(b) Hire Shop A: £4.50 + £5 × 16 = £84.50
Hire Shop B: £16.50 + £4 × 16 = £80.50
Hire Shop B is cheaper by £4
Answer: (a) 12 (b) Hire Shop B, by £4

3 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 20 ÷ 2 = 10 cm
$OA^2 = OM^2 + AM^2 = 24^2 + 10^2 = 676$
OA = 26 cm

(b) sin(angle AOM) = $\frac{10}{26}$

Angle AOM = 22.62°, so angle AOB = 2 × 22.62 = 45.2°
Answer: (a) 26 cm (b) 45.2°

4 Fencing against a wall

A farmer uses 76 m of fencing to enclose a rectangle against a wall, as shown.
(a) Show that the area, A m^2, is $A = 76x - 2x^2$.
(b) Find the maximum area, proving it is a maximum.

Side parallel to the wall = $76 - 2x$, so $A = x(76 - 2x) = -2x^2 + 76x$

$\frac{dA}{dx} = -4x + 76 = 0$ gives $x = 19$

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

$A = 76 \times 19 - 2 \times 19^2 = 722$
Answer: 722 m^2

5 Quadratic inequalities

Solve $6x^2 - 5x - 6 \leq 0$

Factorise: $(3x + 2)(2x - 3) \leq 0$

Critical values: $x = -\frac{2}{3}$ and $x = \frac{3}{2}$

Sketch the U-shaped curve: the expression is negative or zero between the critical values

Answer: $-\frac{2}{3} \leq x \leq \frac{3}{2}$

6 Three-part ratios

Fatima, Isla and Priya share some stickers.
Fatima : Isla = 3 : 8
Isla : Priya = 6 : 4
(a) Work out the ratio Fatima : Isla : Priya.
(b) They share 490 stickers altogether. How many stickers does Fatima get?

(a) Make the Isla part the same: 3 : 8 = 9 : 24 and 6 : 4 = 24 : 16
Fatima : Isla : Priya = 9 : 24 : 16
(b) Total parts = 49, so 1 part = 490 ÷ 49 = 10
Fatima gets 9 × 10 = 90 stickers
Answer: (a) 9 : 24 : 16 (b) 90 stickers

7 Algebraic proof

Show algebraically that the product of $(4m - 1)$ and $(4m + 1)$ is always 1 less than a multiple of 16.

$(4m - 1)(4m + 1) = 16m^2 + 4m - 4m - 1$
$= 16m^2 - 1$
$16m^2$ is a multiple of 16, so the product is 1 less than a multiple of 16
Answer: Proved

8 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where
$s = 24t^{\frac{3}{2}} - 2t^2$ $(t \geq 0)$
Find the velocity and the acceleration of the particle when t = 4.

$v = \frac{ds}{dt} = -4t + 36t^{\frac{1}{2}}$
$a = \frac{dv}{dt} = -4 + 18t^{-\frac{1}{2}}$
When t = 4: v = 56 and a = 5
Answer: v = 56 m/s, a = 5 m/s^2

9 Recurring decimals

Show that $0.8\dot{4} = \frac{38}{45}$

Let x = 0.8444...
10x = 8.4444...
100x = 84.4444...
Subtract: 90x = 76
$x = \frac{76}{90} = \frac{38}{45}$
Answer: x = $\frac{38}{45}$

10 Area of a triangle: ½ab sin C

The area of triangle ABC is 21.9 cm^2.
Angle C is acute. Work out the size of angle C.
Give your answer correct to 1 decimal place. (Calculator)

Area = ½ × a × b × sin C
21.9 = ½ × 14.4 × 10.7 × sin C

$$\sin C = \frac{43.8}{154.08} = 0.2843$$

C = 16.5°

Answer: 16.5°