Day 280 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = 3x^5 - 6x^3 + x^2 + 2$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$3x^5 \to 15x^4$, $-6x^3 \to -18x^2$, $x^2 \to 2x$

$\frac{dy}{dx} = 15x^4 - 18x^2 + 2x$

Answer: $\mathbf{\frac{dy}{dx} = 15x^4 - 18x^2 + 2x}$

2 Rational and irrational numbers

Here are six numbers.

$\sqrt{2} \times \sqrt{6}$, $\sqrt{3} \times \sqrt{5}$, $\frac{1}{\sqrt{3}}$, $(2 + \sqrt{5})(2 - \sqrt{5})$, $\sqrt{50}$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$

Write down the rational numbers.

Simplify each number:

$\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$; $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$; $\sqrt{50} = 5\sqrt{2}$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$

Answer: $\mathbf{(2 + \sqrt{5})(2 - \sqrt{5})}$, $\mathbf{(\sqrt{6} + 2)(\sqrt{6} - 2)}$

3 Exact trigonometric values

Work out the exact value of cos 30° ÷ cos 60°

Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 30° \div \cos 60° = \frac{\sqrt{3}}{2} \div \frac{1}{2}$

$= \sqrt{3}$

Answer: $\mathbf{\sqrt{3}}$

4 Position vector of a point dividing a line

The position vectors of A and B are $\begin{pmatrix}1\\3\end{pmatrix}$ and $\begin{pmatrix}-9\\-7\end{pmatrix}$.

The point P lies on AB so that AP : PB = 3 : 2.

Find the position vector of P.

$\mathbf{AB} = \begin{pmatrix}-9\\-7\end{pmatrix} - \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}-10\\-10\end{pmatrix}$

$\mathbf{AP} = \frac{3}{5}\mathbf{AB} = \begin{pmatrix}-6\\-6\end{pmatrix}$

$\mathbf{OP} = \mathbf{OA} + \mathbf{AP} = \begin{pmatrix}-5\\-3\end{pmatrix}$

Answer: $\mathbf{\begin{pmatrix}-5\\-3\end{pmatrix}}$

5 Comparing costs

Sparkle Cleaners has a call-out fee of £1 plus £4 per hour.

Fresh Start has a call-out fee of £23.50 plus £2.50 per hour.

(a) Work out the number of hours for which the two costs are the same.

(b) Which is cheaper for 23 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $1 + 4n = 23.50 + 2.50n$

$1.50n = 22.50$, so $n = 15$

(b) Sparkle Cleaners: £1 + £4 × 23 = £93

Fresh Start: £23.50 + £2.50 × 23 = £81

Fresh Start is cheaper by £12

Answer: (a) 15 (b) Fresh Start, by £12

6 Similar shapes

Two shapes are mathematically similar.
The smaller has height 40 cm and area 28 cm^2.
The larger has area 10.08 cm^2.
Work out the height of the larger shape.

Ratio of areas = 10.08 : 28 = 9 : 25
Ratio of lengths = 3 : 5
Height = $40 \times \frac{3}{5} = 24$ cm

Answer: 24 cm

7 Second derivative and nature at a given point

$y = -2x^3 - 6x^2 + 9$
Use the second derivative to show that the curve has a maximum at x = 0.

$\frac{dy}{dx} = -6x^2 - 12x$

When x = 0: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = -12x - 12$

When x = 0: $\frac{d^2y}{dx^2} = -12$, which is negative

So there is a maximum at x = 0
Answer: maximum at x = 0

8 Finding the rate of interest

Isla invested £1,000 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £1,060.90.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $1000 \times m^2 = 1060.90$
$m^2 = 1060.90 \div 1000 = 1.0609$
$m = \sqrt{1.0609} = 1.03$
Rate = $(1.03 - 1) \times 100 = 3\%$
Answer: 3% per year

9 No stationary points

A curve has equation $y = 2x^3 - 6x^2 + 12x$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 - 12x + 12 = 6(x - 1)^2 + 6$

A squared term is never negative, so $\frac{dy}{dx} \geq 6 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 6 (when x = 1)
Answer: (b) 6

10 Solving AX = B

$A = \begin{pmatrix} -4 & 1 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -12 \\ 24 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = -\frac{1}{9}\begin{pmatrix} 1 & -1 \\ -5 & -4 \end{pmatrix}$

$X = -\frac{1}{9}\begin{pmatrix} 1 & -1 \\ -5 & -4 \end{pmatrix}\begin{pmatrix} -12 \\ 24 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

Answer: $\mathbf{X} = \begin{pmatrix} \mathbf{4} \\ \mathbf{4} \end{pmatrix}$