Day 28 **IGCSE 10-a-day: Model Solutions**

1 Stationary points of a cubic

A curve has equation $y = 2x^3 - 9x^2 - 24x + 5$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = 6x^2 - 18x - 24$

Stationary points: $\frac{dy}{dx} = 0$, so $6(x + 1)(x - 4) = 0$

$x = -1$ or $x = 4$
$y = 18$ when $x = -1$ and $y = -107$ when $x = 4$
Answer: $(-1, 18)$ and $(4, -107)$

2 Inverse functions

$f(x) = \frac{x + 3}{x + 5}$

(a) Find $f^{-1}(x)$.
(b) Find the value of $f^{-1}(3)$.

(a) Let $y = \frac{x + 3}{x + 5}$. Then $y(x + 5) = x + 3$
$xy + 5y = x + 3$
Collect x terms: $xy - x = 3 - 5y$
Factorise: $x(y - 1) = -5y + 3$, so $x = \frac{-5y + 3}{y - 1}$
$f^{-1}(x) = \frac{-5x + 3}{x - 1}$
(b) $f^{-1}(3) = \frac{(-5) \times 3 + 3}{1 \times 3 - 1} = \frac{-12}{2} = -6$
Answer: (a) $f^{-1}(x) = \frac{-5x + 3}{x - 1}$ (b) -6

3 Divide then differentiate

$y = \frac{-7x^5 + 6x^3 + 5x^2}{x^2}$

Find $\frac{dy}{dx}$.

Divide each term by x^2: $y = -7x^3 + 6x + 5$
$-7x^3 \to -21x^2$, $6x \to 6$
$\frac{dy}{dx} = -21x^2 + 6$
Answer: $\frac{dy}{dx} = -21x^2 + 6$

4 Parallel vectors

A = $(-3, 2)$, B = $(2, -3)$, C = $(1, -2)$ and D = $(k, -12)$.
AB is parallel to **CD**.
Find the value of k.

$\mathbf{AB} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ and $\mathbf{CD} = \begin{pmatrix} k - 1 \\ -10 \end{pmatrix}$
Vertical parts: $-10 = \lambda \times (-5)$, so $\lambda = 2$
Horizontal: $k - 1 = 2 \times 5 = 10$
$k = 11$
Answer: $k = 11$

5 Increasing or decreasing at a point

$y = -x^3 + 12x - 8$

At the point where $x = 1$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = -3x^2 + 12$

When $x = 1$: $\frac{dy}{dx} = 9$

9 is positive, so the curve is increasing

Answer: increasing (gradient 9)

6 Combined and changing means

The mean mark of 14 students in Group X is 57.
The mean mark of 11 students in Group Y is 65.
Work out the mean mark of all 25 students.

Total marks for Group X = 14 × 57 = 798
Total marks for Group Y = 11 × 65 = 715
Mean = (798 + 715) ÷ 25 = 60.52

Answer: 60.52

7 Rationalising the denominator

Write $\frac{18}{2+\sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are integers.

Multiply top and bottom by $(2 - \sqrt{7})$:

$\frac{18}{2+\sqrt{7}} = \frac{18(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$

Denominator: $4 - 7 = -3$

$= -12 + 6\sqrt{7}$

Answer: $-12 + 6\sqrt{7}$

8 Area and quadratic equations

The diagram shows a rectangle of length $(3x - 1)$ cm, width $(x + 2)$ cm and area 98 cm^2.
(a) Show that $3x^2 + 5x - 100 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(3x - 1)(x + 2) = 98$

Expand: $3x^2 + 5x - 2 = 98$, so $3x^2 + 5x - 100 = 0$

(b) Factorise: $(x - 5)(3x + 20) = 0$

$x = 5$ ($x = -\frac{20}{3}$ is not possible as a length would be negative)

Length = 14 cm and width = 7 cm
Perimeter = 2 × (14 + 7) = 42 cm

Answer: (b) 42 cm

9 Ratio and money transferred

Uma and Maya share some money in the ratio 7 : 5.
Uma gives Maya £95.
The ratio of their money is now 1 : 5.
How much money do they have altogether?

Let the amounts be 7k and 5k, so the total is 12k

Uma now has $7k - 95$, which is $\frac{1}{6}$ of the total

$6(7k - 95) = 1 \times 12k$

$42k - 570 = 12k$, so $30k = 570$ and $k = 19$

Total = 12 × 19 = £228

Answer: £228

10 Percentages with an unknown rate

The price of a ticket is decreased by x%.
The new price is then increased by x%.
The final price is 36% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 36% less, so the multiplier is 0.64: $1 - \frac{x^2}{10000} = 0.64$

$\frac{x^2}{10000} = 0.36$, so $x^2 = 3600$ and $x = 60$

Answer: x = 60