Day 279 **IGCSE 10-a-day: Model Solutions**

1 Powers of standard form

Work out $(1.5 \times 10^3)^3$
Give your answer in standard form.

$(1.5 \times 10^3)^3 = 1.5^3 \times (10^3)^3$
$1.5^3 = 3.375$ and $(10^3)^3 = 10^9$
$= 3.375 \times 10^9$
Answer: 3.375×10^9

2 Variation in context

The braking distance d metres of a car is directly proportional to the square of its speed v km/h.
When v = 2, d = 8m.
Find the value of d when v = 3.

$d = kv^2$
$8 = k \times 2^2$ so $k = 2$
$d = 2v^2$
$d = 2 \times 9 = 18$
Answer: 18m

3 Rate of change of area or volume

The volume, V cm^3, of a sphere of radius r cm is $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{dV}{dr}$ when r = 8.

(b) Find the value of r when $\frac{dV}{dr} = 144\pi$.

$\frac{dV}{dr} = 4\pi r^2$

When r = 8: $\frac{dV}{dr} = 256\pi$

$4\pi r^2 = 144\pi$, so $4r^2 = 144$ and r = 6 (lengths are positive)
Answer: (a) 256π (b) r = 6

4 Median and quartiles from a list

Here are 15 numbers.
58, 46, 18, 23, 47, 30, 11, 22, 4, 30, 5, 5, 39, 57, 36
Find (a) the median (b) the interquartile range.

Ordered: 4, 5, 5, 11, 18, 22, 23, 30, 30, 36, 39, 46, 47, 57, 58
(a) Median is the 8th value = 30
(b) Lower quartile is the 4th value = 11; upper quartile is the 12th value = 46
IQR = 46 − 11 = 35
Answer: (a) 30 (b) 35

5 Tangents parallel to a line

A curve has equation $y = x^3 - 12x^2 + 41x - 5$.
Find the equations of the two tangents to the curve that are parallel to the line y = 5x + 9.

Parallel lines have equal gradients: gradient = 5
$\frac{dy}{dx} = 3x^2 - 24x + 41 = 5$, so $3x^2 - 24x + 36 = 0$
$3(x - 2)(x - 6) = 0$ gives x = 2 or x = 6
At x = 2: y = 37, tangent $y - 37 = 5(x - 2)$
At x = 6: y = 25, tangent $y - 25 = 5(x - 6)$
Answer: $y = 5x + 27$ and $y = 5x - 5$

6 Speed-time graphs

The graph shows the speed of a train during 20 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 20 seconds.

(a) Acceleration = change in speed ÷ time = (20 − 8) ÷ 4 = 3 m/s^2

(b) Distance = area under the graph

Trapezium: $\frac{8+20}{2} \times 4 = 56$; rectangle: $20 \times 10 = 200$; triangle: $\frac{1}{2} \times 6 \times 20 = 60$

Total = 316 m, so average speed = 316 ÷ 20 = 15.8 m/s

Answer: (a) 3 m/s^2 (b) 15.8 m/s

7 Bounds of area and perimeter

A rectangle has length 23 cm and width 17 cm.
Both measurements are correct to the nearest centimetre.
Jess says that the area of the rectangle could be exactly 367 cm^2.
Is Jess correct? You must show your working.

$22.5 \leq$ length < 23.5 and $16.5 \leq$ width < 17.5
Lower bound of area = $22.5 \times 16.5 = 371.25$
Upper bound of area = $23.5 \times 17.5 = 411.25$
So $371.25 \leq$ area < 411.25
367 is outside this interval, so Jess is not correct

Answer: No, 367 is outside the interval

8 Iteration: rearranging equations

(a) Show that the equation $x^3 - x - 14 = 0$ can be rearranged to give $x = \sqrt[3]{x + 14}$ (Calculator)
(b) Use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 14}$ with $x_0 = 2$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 - x - 14 = 0$: $x^3 = x + 14$

Take the cube root of both sides: $x = \sqrt[3]{x + 14}$

(b) $x_1 = \sqrt[3]{2 + 14} = 2.51984...$

$x_2 = \sqrt[3]{x_1 + 14} = 2.54684...$

$x_1 = 2.520$ and $x_2 = 2.547$ (3 d.p.)

Answer: (b) $x_1 = 2.520$, $x_2 = 2.547$

9 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 1. N is on OB with ON : NB = 2 : 3. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{2}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{2}\mathbf{a} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{2}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{2}{5}t\mathbf{b}$

Equate: $\frac{1}{2}s = 1 - t$ and $1 - s = \frac{2}{5}t$

Solving: $s = \frac{3}{4}$ and $t = \frac{5}{8}$

OX = $\frac{3}{8}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: (a) $\frac{1}{2}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{3}{8}\mathbf{a} + \frac{1}{4}\mathbf{b}$

10 Vertical distance between stationary points

The curve $y = 2x^3 + 6x^2 - 18x - 3$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 + 12x - 18 = 6(x + 3)(x - 1)$

$x = -3$ and $x = 1$

At $x = -3$, $y = 51$; at $x = 1$, $y = -13$

Difference $= 51 - (-13) = 64$

Answer: 64