Day 278 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = -2x^3 - \frac{5}{x^3}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -2x^3 - 5x^{-3}$

$-2x^3 \rightarrow -6x^2$, $-5x^{-3} \rightarrow 15x^{-4}$

$\frac{dy}{dx} = -6x^2 + 15x^{-4}$

Answer: $\mathbf{\frac{dy}{dx} = -6x^2 + \frac{15}{x^4}}$

2 Tests of divisibility

The 4-digit number 397k is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 3 + 9 + 7 + k = 19 + k
This must be a multiple of 9, so k = 8
Answer: k = 8

3 Column vector arithmetic

$\mathbf{a} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

(a) Work out $\mathbf{a} + \mathbf{b}$.
(b) Work out $4\mathbf{a} - 2\mathbf{b}$.

(a) $\begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$

(b) $4\mathbf{a} = \begin{pmatrix} 8 \\ 28 \end{pmatrix}$ and $2\mathbf{b} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 8 \\ 28 \end{pmatrix} - \begin{pmatrix} 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 24 \end{pmatrix}$

Answer: (a) $\mathbf{\begin{pmatrix} 6 \\ 9 \end{pmatrix}}$ **(b)** $\mathbf{\begin{pmatrix} 0 \\ 24 \end{pmatrix}}$

4 Pressure

A wooden block in the shape of a cuboid weighs 90 N.
Its smallest face has area 150 cm^2. Its largest face has area 250 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the wooden block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 250 cm^2 = 0.025 m^2
(a) Greatest pressure is on the smallest face: 90 ÷ 0.015 = 6,000 N/m^2
(b) Least pressure is on the largest face: 90 ÷ 0.025 = 3,600 N/m^2
Answer: (a) 6,000 N/m^2 (b) 3,600 N/m^2

5 Similar shapes

Two shapes are mathematically similar.
The smaller has height 2 cm and area 28 cm^2.
The larger has area 112 cm^2.
Work out the height of the larger shape.

Ratio of areas = 112 : 28 = 4 : 1
Ratio of lengths = 2 : 1
Height = 2 × 2 = 4 cm
Answer: 4 cm

6 Fractional indices

Given that $x > 0$, solve $x^{\frac{1}{3}} = 5$

Raise both sides to the power 3

$x = (5)^3$

$x = 125$

Answer: x = 125

7 Solving trigonometric equations

Solve $2\tan x + 2 = 0$ for $0 \leq x \leq 360$.

$\tan x = -1$

Principal value: $x = \tan^{-1}(-1) = -45°$

Other solutions in the range: use symmetry of the tan graph

$x = 135, 315$

Answer: x = 135, 315

8 Stationary point of $y = ax + b/\sqrt{x}$

A curve has equation $y = 3x + \frac{48}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 3 - 24x^{-3/2}$

$\frac{dy}{dx} = 0$: $24x^{-3/2} = 3$, so $x^{3/2} = 8$

$x = 8^{2/3} = 4$, and $y = 36$

$\frac{d^2y}{dx^2} = 36x^{-5/2}$

At $x = 4$, $\frac{d^2y}{dx^2} = \frac{9}{8}$, which is positive, so a minimum

Answer: (4, 36), minimum

9 Tangent to a circle: proof

Show that the line $y = -2x + 20$ is a tangent to the circle $x^2 + y^2 = 80$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -2x + 20$ into $x^2 + y^2 = 80$: $x^2 + (-2x + 20)^2 = 80$

Expand: $5x^2 - 80x + 320 = 0$

$b^2 - 4ac = (-80)^2 - 4 \times 5 \times 320 = 6400 - 6400 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-(-80)}{2 \times 5} = 8$, and $y = -2 \times 8 + 20 = 4$. The point is (8, 4)

Answer: (8, 4)

10 Distance travelled

The diagram shows the graph of $s = -t^3 + 6t^2 - 9t$, the displacement in metres of a particle P from O after t seconds.
P changes direction at A and B. Work out the total distance travelled in the first 5 seconds.

$v = \frac{ds}{dt} = -3t^2 + 12t - 9 = 0$ at $t = 1$ and $t = 3$ (A and B)

$s(0) = 0$, $s(1) = -4$, $s(3) = 0$, $s(5) = -20$

Distances: $|-4 - 0| + |0 - (-4)| + |-20 - 0|$

$= 4 + 4 + 20 = 28$

Answer: 28 m