Day 277 **IGCSE 10-a-day: Model Solutions**

1 Multiplying and dividing surds

Simplify fully $5\sqrt{10} \times 3\sqrt{14}$

$5\sqrt{10} \times 3\sqrt{14} = 15\sqrt{140}$

$\sqrt{140} = 2\sqrt{35}$, so $15 \times 2\sqrt{35} = 30\sqrt{35}$

Answer: $30\sqrt{35}$

2 Trigonometric graphs: amplitude and period

For $0 \leq x \leq 360$, a graph has equation

$y = 3\cos 3x° - 3$

Write down

(a) the smallest positive value of x at which y is greatest

(b) the greatest value of y

(a) cos 3x is 1 first when $3x = 360°$ (x = 0 is not positive), so $x = 360 \div 3 = 120$

(b) Greatest value = $3 + (-3) = 0$

Answer: (a) 120 (b) 0

3 Inequalities

(a) Solve $9 - 2x \geq 11$

(b) n is an integer. Write down all the values of n that satisfy

$4 \leq 4n - 4 \leq 32$

(a) Subtract 9: $-2x \geq 2$

Divide by -2 and reverse the inequality sign: $x \leq -1$

(b) Add 4: $8 \leq 4n \leq 36$

Divide by 4: $2 \leq n \leq 9$

Integers: 2, 3, 4, 5, 6, 7, 8, 9

Answer: (a) $x \leq -1$ (b) 2, 3, 4, 5, 6, 7, 8, 9

4 Minimum gradient of a curve

A curve has equation $y = 2x^3 - 12x^2 + 36x + 4$.

Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 6x^2 - 24x + 36$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $12x - 24 = 0$, so $x = 2$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is $12 > 0$), so this is a minimum

$\frac{dy}{dx} = 12$ when $x = 2$

Answer: minimum gradient 12 when x = 2

5 Percentage change

The population of a village decreased by 10% in 2023.

In 2024 the population decreased by a further 15%.

Work out the overall percentage decrease over the two years.

Multipliers: 0.9 and 0.85

Overall multiplier = $0.9 \times 0.85 = 0.765$

$0.765 - 1 = -0.235$, so an overall decrease of 23.5%

Answer: 23.5% decrease

6 Probabilities with an unknown

A biased four-sided dice lands on one of four outcomes with these probabilities.
1: 3x; 2: x; 3: 2x; 4: 0.7
(a) Work out the value of x.
(b) The biased four-sided dice is used 40 times. Work out an estimate for the number of times it lands on 1.

(a) Probabilities add up to 1: 3x + x + 2x + 0.7 = 1
6x + 0.7 = 1, so 6x = 0.3
x = 0.05
(b) P(1) = 3 × 0.05 = 0.15, so 40 × 0.15 = 6
Answer: (a) x = 0.05 (b) 6

7 Rate of cooling or growth

The temperature, T °C, of a metal bar t minutes after it is removed from a heat source is modelled by

$T = 25 + \frac{32}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 4.

(b) Find the value of t when the temperature is falling at 8 °C per minute.

$T = 25 + 32t^{-1}$, so $\frac{dT}{dt} = -32t^{-2}$

When t = 4: $\frac{dT}{dt} = -2$

$-\frac{32}{t^2} = -8$, so $t^2 = 4$ and t = 2

Answer: (a) −2 °C/min (b) t = 2

8 Loci and regions

ABCD is a rectangle with AB = 17 cm and AD = 15 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 12^2 = 18\pi$ cm^2

Answer: 18π cm^2

9 Bearings with sine and cosine rules

A ship sails 8 km from P to Q on a bearing of 075°.
It then sails 5 km from Q to R on a bearing of 168°.
(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 075° + 180° − 168° = 87°
$PR^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 87°$ so PR = 9.21 km

$\cos QPR = \frac{8^2 + 9.2094^2 - 5^2}{2 \times 8 \times 9.2094}$, angle QPR = 32.83°

Bearing of R from P = 075° + 32.83° = 108°
Answer: (a) 9.21 km (b) 108°

10 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where

$s = t^3 - 3t^2 + 4t - 3$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 3t^2 - 6t + 4 = 3(t - 1)^2 + 1$

A square is never negative, so $v \geq 1 > 0$ and P is never at rest

Least value of v occurs when t = 1, giving 1

Answer: (b) 1 m/s when t = 1 s