Day 276 **IGCSE 10-a-day: Model Solutions**

1 Stationary points and their nature

$y = x^3 + 3x^2 + 4$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 + 6x = 3x(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$ or $x = 0$, so the points are $(-2, 8)$ and $(0, 4)$

$\frac{d^2y}{dx^2} = 6x + 6$

$x = -2$: $\frac{d^2y}{dx^2} = -6$, which is negative, so a maximum

$x = 0$: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

Answer: (a) $(-2, 8)$ and $(0, 4)$ (b) $(-2, 8)$ is a maximum, $(0, 4)$ is a minimum

2 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix}5\\4\end{pmatrix}$, $\mathbf{q} = \begin{pmatrix}-4\\2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}6\\6\end{pmatrix}$.

Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix}-8\\4\end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}-8\\4\end{pmatrix} - \begin{pmatrix}6\\6\end{pmatrix} = \begin{pmatrix}-9\\2\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{-9}\\\mathbf{2}\end{pmatrix}$

3 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = t^3 + t^2 + 5t + 5$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when $t = 1$.

$v = \frac{ds}{dt} = 3t^2 + 2t + 5$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t + 2$

When $t = 1$: $v = 10$ and $a = 8$

Answer: (a) 10 m/s (b) 8 m/s^2

4 HCF and LCM

(a) Write 378 and 6 as products of prime factors.

(b) Find the HCF and the LCM of 378 and 6.

Give your answers as numbers.

(a) $378 = 2 \times 3^3 \times 7$ and $6 = 2 \times 3$

(b) HCF $= 2 \times 3 = 6$

LCM $= 2 \times 3^3 \times 7 = 378$

Answer: (a) $2 \times 3^3 \times 7$; 2×3 (b) HCF = 6, LCM = 378

5 Trigonometric graphs

The diagram shows the graph of $y = a \sin x + c$.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when $x = 330°$.

(a) The maximum value is 6 and the minimum value is 0
$a = (6 - (0)) \div 2 = 3$
$c = (6 + (0)) \div 2 = 3$

(b) $y = 3 \sin 330° + 3$, and $\sin 330° = -\frac{1}{2}$

$y = \frac{3}{2}$

Answer: (a) a = 3, c = 3 (b) $y = \frac{3}{2}$

6 Collinear points with vectors

$\mathbf{OA} = 2\mathbf{a} - 3\mathbf{b}$, $\mathbf{OB} = 8\mathbf{a} - 11\mathbf{b}$ and $\mathbf{OC} = -\mathbf{a} + \mathbf{b}$.
Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = 6\mathbf{a} - 8\mathbf{b}$
$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -3\mathbf{a} + 4\mathbf{b}$

$\mathbf{AC} = -\frac{1}{2}\mathbf{AB}$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: $\mathbf{AC} = -\frac{1}{2}\mathbf{AB}$, so A, B, C are collinear

7 Vertical distance between stationary points

The curve $y = x^3 + 6x^2 + 9x - 8$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 3x^2 + 12x + 9 = 3(x + 3)(x + 1)$
$x = -3$ and $x = -1$
At $x = -3$, $y = -8$; at $x = -1$, $y = -12$
Difference $= -8 - (-12) = 4$
Answer: 4

8 Algebraic fractions: adding and subtracting

Show that $\frac{3}{x-5} - \frac{2}{x+2}$ can be written as $\frac{x+16}{(x-5)(x+2)}$

The common denominator is $(x - 5)(x + 2)$

$$\frac{3}{x-5} - \frac{2}{x+2} = \frac{3(x+2) - 2(x-5)}{(x-5)(x+2)}$$

Expand the numerator: $x + 16$

$$\frac{x+16}{(x-5)(x+2)}$$

Answer: $\frac{x+16}{(x-5)(x+2)}$

9 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 12 cm and the area of triangle ADE is 81 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 12 = 21 cm

Length scale factor = $21 \div 9 = \frac{7}{3}$

Area scale factor = $(\frac{7}{3})^2 = \frac{49}{9}$

Area of triangle ABC = $81 \times \frac{49}{9} = 441$ cm^2

Trapezium = $441 - 81 = 360$ cm^2

Answer: 360 cm^2

10 Combining means

The mean of the heights in cm for the men is 72.
The mean of the heights in cm for the women is 67.
The mean for all of the men and women together is 71.
Work out the ratio of the number of men to the number of women.

Let there be p men and q women
Total = 72p + 67q, and the overall mean gives 72p + 67q = 71(p + q)
1p = 4q
p : q = 4 : 1
Answer: 4 : 1