Day 275 **IGCSE 10-a-day: Model Solutions**

1 Tangent to a curve with negative or fractional powers

A curve has equation $y = 4x^{3/2} - 2\sqrt{x}$.
Find the equation of the tangent to the curve at the point where $x = 1$.

$\frac{dy}{dx} = 6\sqrt{x} - x^{-1/2}$
At $x = 1$: $y = 2$ and gradient $= 5$
$y - 2 = 5(x - 1)$
$y = 5x - 3$
Answer: $y = 5x - 3$

2 Inverse of a linear function

$f(x) = -2x - 8$.
(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(-1)$.

(a) Let $y = -2x - 8$, then rearrange to make x the subject
$x = \frac{-8 - y}{2}$, so $f^{-1}(x) = \frac{-8 - x}{2}$

(b) $f^{-1}(-1) = -\frac{7}{2}$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{-8 - x}{2}}$ (b) $\mathbf{-\frac{7}{2}}$

3 Error intervals

The volume of juice is 820 litres, correct to 2 significant figures.
Write down the error interval for the volume of juice, using v.

The value has been rounded to the nearest 10
Half of this is 5, so go 5 below and 5 above
$820 - 5 = 815$ and $820 + 5 = 825$
$815 \leq v < 825$
Answer: $\mathbf{815 \leq v < 825}$

4 Probability with sets

$n(\xi) = 100$, $n(A) = 43$, $n(B) = 43$ and $n(A \cap B) = 40$.
Find $P(A \mid B)$ and $P(B \mid A')$.

$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{40}{43}$
$n(A') = 100 - 43 = 57$ and $n(B \cap A') = 43 - 40 = 3$
$P(B \mid A') = \frac{3}{57} = \frac{1}{19}$

Answer: $\mathbf{P(A \mid B) = \frac{40}{43}}$, $\mathbf{P(B \mid A') = \frac{1}{19}}$

5 Fractions removed from a ratio

The ratio of red to blue marbles in a jar is 9 : 8.
Then $\frac{1}{3}$ of the red and $\frac{3}{4}$ of the blue are taken out.
Work out the ratio of red to blue now.

Left: $\frac{2}{3} \times 9 = 6$ of the red part, $\frac{1}{4} \times 8 = 2$ of the blue part
6 : 2
Multiply by 1: 6 : 2 = 3 : 1
Answer: 3 : 1

6 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 6)$ cm, $(x - 4)$ cm and $(x - 2)$ cm.
The longest side is $(x - 2)$ cm.
(a) Show that $x^2 - 16x + 48 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 6)^2 + (x - 4)^2 = (x - 2)^2$
Expand: $x^2 - 12x + 36 + x^2 - 8x + 16 = x^2 - 4x + 4$
Rearrange: $x^2 - 16x + 48 = 0$
(b) Factorise: $(x - 12)(x - 4) = 0$, so $x = 12$ (the other root, $x = 4$, does not give valid side lengths)
The shorter sides are 6 cm and 8 cm, so area $= \frac{1}{2} \times 6 \times 8 = 24$ cm^2

Answer: (b) 24 cm^2

7 Where is the second derivative zero?

$y = x^3 + 9x^2 + 5x + 4$
(a) Find $\frac{d^2y}{dx^2}$.
(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 3x^2 + 18x + 5$
$\frac{d^2y}{dx^2} = 6x + 18$
$\frac{d^2y}{dx^2} = 0$ gives $6x + 18 = 0$, so $x = -3$
$\frac{dy}{dx} = -22$ when $x = -3$
Answer: (a) $6x + 18$ (b) $x = -3$, gradient -22

8 Rearranging when the subject appears twice

Make r the subject of $P = \frac{4r - 2}{3r - 1}$

Multiply by $3r - 1$: $P(3r - 1) = 4r - 2$
Expand: $3rP - P = 4r - 2$
Collect the r terms: $3rP - 4r = -2 + P$
Factorise: $r(3P - 4) = P - 2$
$r = \frac{P - 2}{3P - 4}$

Answer: $r = \frac{P - 2}{3P - 4}$

9 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by
$N = -t^3 + 9t^2 + 10t + 100$ for $0 \leq t \leq 12$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 18t + 10$
Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 18 = 0$, so $t = 3$
$\frac{d^2N}{dt^2}$ is decreasing (gradient -6), so the rate is a maximum
$\frac{dN}{dt} = 37$ when $t = 3$
Answer: t = 3, rate 37 per hour

10 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $5\mathbf{a} - 3\mathbf{b}$, OQ = $9\mathbf{a}$ and OR = $17\mathbf{a} + 6\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $4\mathbf{a} + 3\mathbf{b}$
QR = OR − OQ = $8\mathbf{a} + 6\mathbf{b}$
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 1 : 2
Answer: (b) 1 : 2