Day 273 **IGCSE 10-a-day: Model Solutions**

1 Expanding three brackets

Expand and simplify $(x - 1)(x + 5)(x - 5)$

Expand the first two brackets: $(x - 1)(x + 5) = x^2 + 4x - 5$

Multiply by $(x - 5)$: $x(x^2 + 4x - 5) - 5(x^2 + 4x - 5)$

$= x^3 - x^2 - 25x + 25$

Answer: $x^3 - x^2 - 25x + 25$

2 Given a stationary point, find k

The curve $y = x^3 + 6x^2 + kx$ has a stationary point at $x = -1$.

(a) Find the value of k.

(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + 12x + k$

$x = -1$: $-9 + k = 0$, so $k = 9$

$\frac{dy}{dx} = 3x^2 + 12x + 9 = 3(x + 1)(x + 3)$

Other root: $x = -3$

$y = 0$

Answer: (a) k = 9 (b) (−3, 0)

3 Multiplying and dividing surds

Simplify fully $\frac{3\sqrt{32}}{2\sqrt{2}}$

$\sqrt{32} = 4\sqrt{2}$

$\frac{3\sqrt{32}}{2\sqrt{2}} = \frac{12\sqrt{2}}{2\sqrt{2}}$

$= 6$

Answer: 6

4 Gradient from a tangent

The diagram shows part of the curve $y = -x^2 - x + 6$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)

(a) Use the two marked points on the tangent to work out its gradient.

(b) Use differentiation to find the x coordinate of P.

Gradient $= \frac{7 - 4}{0 - (-3)} = 1$

$\frac{dy}{dx} = -2x - 1$

$-2x - 1 = 1$, so $x = -1$

Answer: (a) 1 (b) x = −1

5 Image of a point under a graph transformation

The point P(8, −8) lies on the graph of $y = f(x)$.

Write down the coordinates of the image of P on the graph of

$y = 3f(x - 1) + 4$

x increases by 1: $8 + 1 = 9$

y: multiply by 3 then add 4: $-8 \times 3 + 4 = -20$

Answer: (9, −20)

6 Volume and surface area of composite solids

A solid is made from a hemisphere joined to a cone. Both have radius 8 cm and the slant height of the cone is 10 cm.
Work out the total surface area of the solid. Give your answer in terms of π.

Curved surface area of cone = $\pi \times r \times l = \pi \times 8 \times 10 = 80\pi$

Surface area of hemisphere = $2\pi r^2 = 2 \times \pi \times 8^2 = 128\pi$

Total = $80\pi + 128\pi = 208\pi$ cm^2

(The flat circle where the two solids join is not part of the surface.)

Answer: 208π cm^2

7 Stationary points of y = ax + b/x

$y = x + \frac{16}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 1 - \frac{16}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{16}{x^2} = 1$, so $x^2 = 16$ and $x = 4$ or $x = -4$

$y = 8$ when $x = 4$ and $y = -8$ when $x = -4$

$\frac{d^2y}{dx^2} = \frac{32}{x^3}$

$x = 4$: $\frac{d^2y}{dx^2} = \frac{1}{2}$ so minimum; $x = -4$: $\frac{d^2y}{dx^2} = -\frac{1}{2}$ so maximum

Answer: (4, 8) minimum; (−4, −8) maximum

8 Reading a trigonometric graph

The diagram shows the graph of $y = a \cos bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 6, least value = 0

c = ½(6 + 0) = 3 and a = ½(6 − 0) = 3

One full cycle takes 360° (1 cycle between 0 and 360), so b = 360 ÷ 360 = 1

Answer: a = 3, b = 1, c = 3

9 Surds with unknowns

Given that $(a + \sqrt{13})(b + \sqrt{13}) = 37 + 11\sqrt{13}$,
where a and b are positive integers with a < b, find the values of a and b.

$(a + \sqrt{13})(b + \sqrt{13}) = ab + (a + b)\sqrt{13} + 13$

Compare the $\sqrt{13}$ terms: $a + b = 11$

Compare the integer terms: $ab + 13 = 37$, so $ab = 24$

Two numbers with sum 11 and product 24 are 3 and 8

Answer: a = 3, b = 8

10 Ratio and mixtures

Container A holds juice and water in the ratio 1 : 6.
Container B holds juice and water in the ratio 6 : 3.
21 litres from container A is mixed with some of container B.
The ratio of juice to water in the mixture is 13 : 12.
Work out how many litres were taken from container B.

Let y litres be taken from B

juice: $3 + \frac{6y}{9}$

water: $18 + \frac{3y}{9}$

Ratio 13 : 12, so $12(3 + \frac{6y}{9}) = 13(18 + \frac{3y}{9})$

$(\frac{11}{3})y = 198$, so $y = 54$

Answer: 54 litres