Day 272 **IGCSE 10-a-day: Model Solutions**

1 Integers satisfying a double inequality

n is an integer.
Work out the sum of all the values of n that satisfy
$-2 < -n - 3 < 4$

Add 3 from each part: $1 < -n < 7$
Divide each part by -1 (reverse the signs): $-7 < n < -1$
Integers: $-6, -5, -4, -3, -2$
Sum = -20
Answer: -20

2 Trigonometric graphs

The diagram shows the graph of $y = a \sin bx$, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when $x = 255°$.

(a) The maximum value is 5, so $a = 5$
The graph completes 2 full cycles between 0° and 360°, so $b = 2$
(b) $y = 5 \sin (2 \times 255°) = 5 \sin 510°$
$\sin 150° = \frac{1}{2}$, so $y = \frac{5}{2}$

Answer: (a) $a = 5$, $b = 2$ (b) $y = \frac{5}{2}$

3 Square roots as powers

$y = -8x^{5/2} - 2x^{3/2}$
Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -8x^{5/2} - 2x^{3/2}$
$-8x^{5/2} \rightarrow -20x^{3/2}$, $-2x^{3/2} \rightarrow -3x^{1/2}$
$\frac{dy}{dx} = -20x^{3/2} - 3x^{1/2}$

Answer: $\frac{dy}{dx} = -20x^{3/2} - 3x^{1/2}$

4 Completing the square

(a) Write $x^2 + 10x + 30$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 10x + 30$

(a) Half of 10 is 5, so $x^2 + 10x = (x + 5)^2 - 25$
$x^2 + 10x + 30 = (x + 5)^2 - 25 + 30$
$= (x + 5)^2 + 5$
(b) The minimum is when $x + 5 = 0$, so $x = -5$ and $y = 5$
Answer: (a) $(x + 5)^2 + 5$ (b) $(-5, 5)$

5 Tangent parallel to a given line

The tangents at two points A and B on the curve $y = 2x^3 - 3x^2 - 6x - 6$ are parallel to the line $2y + 12x = 7$.
Find the coordinates of A and B.

Gradient of the line is -6, so $\frac{dy}{dx} = -6$
$6x^2 - 6x - 6 = -6$, so $6x^2 - 6x = 0$ ($\div 6$: $x^2 - x = 0$)
$x(x - 1) = 0$ so $x = 0$ or $x = 1$
A(0, -6) and B(1, -13)
Answer: (0, -6) and (1, -13)

6 Volume and surface area of composite solids

A solid is made from a cone of height 11 cm and a hemisphere. Both have radius 4 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 4^2 \times 11 = \frac{176}{3}\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 4^3 = \frac{128}{3}\pi$

Total = $\frac{176}{3}\pi + \frac{128}{3}\pi = \frac{304}{3}\pi$ cm^3

Answer: $\frac{304}{3}\pi$ cm^3

7 Reverse percentages

The price of a sofa is increased by 50%.
Later the new price is reduced by 15% in a sale.
The sale price is £752.25.
Work out the price before the increase. (Calculator)

Multipliers: 1.5 and 0.85
Overall multiplier = 1.5 × 0.85 = 1.275
Original × 1.275 = £752.25
Original = £752.25 ÷ 1.275 = £590
Answer: £590

8 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx + 4$ has stationary points at $x = -4$ and $x = -2$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = -4$: $48 + -8a + b = 0$. $x = -2$: $12 + -4a + b = 0$
Subtract the equations: $36 + -4a = 0$, so $a = 9$
Then $b = 24$
Answer: $a = 9$, $b = 24$

9 Laws of indices

Write $\frac{16^{n-2} \times 4^{n-2}}{4^n}$ in the form 2^{an+c}

where a and c are integers.

$16 = 2^4$, $4 = 2^2$, $4 = 2^2$
$16^{n-2} = 2^{4(n-2)}$, $4^{n-2} = 2^{2(n-2)}$, $4^n = 2^{2n}$
Exponent: $4(n - 2) + 2(n - 2) - 2n = 4n - 12$
Answer: $a = 4$, $c = -12$

10 Standard form: adding and subtracting

Work out $1.4 \times 10^{-4} + 4.8 \times 10^{-6}$.
Give your answer in standard form.

Write both numbers with the same power of 10, 10^{-6}:
$140 \times 10^{-6} + 4.8 \times 10^{-6}$
$= (140 + 4.8) \times 10^{-6} = 144.8 \times 10^{-6}$
$= 1.448 \times 10^{-4}$
Answer: 1.448×10^{-4}