**Day 271** **IGCSE 10-a-day: Model Solutions**

## 1 Time zones

A flight leaves London at 07:05 (London time).
It takes 7 hours 10 minutes.
When it is 12:00 in London it is 16:00 in Dubai.
At what time, in Dubai time, does the flight land? Say if it is on a different day.

Arrival in London time = 07:05 + 7 h 10 min = 14:15
Dubai is 4 hours ahead of London, so add 4 hours
18:15
**Answer: 18:15 the same day**

## 2 Inverse of a linear function

$f(x) = \frac{x + 5}{3}$.

(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(-1)$.

(a) Let $y = \frac{x + 5}{3}$, then rearrange to make x the subject

$x = 3y - 5$, so $f^{-1}(x) = 3x - 5$
(b) $f^{-1}(-1) = -8$
**Answer: (a) $f^{-1}(x) = 3x - 5$ (b) $-8$**

## 3 Divide then differentiate

$y = \frac{5x^5 - 7x^2 + 7x}{x^2}$

Find $\frac{dy}{dx}$.

Divide each term by $x^2$: $y = 5x^3 - 7 + 7x^{-1}$
$5x^3 \rightarrow 15x^2$, $7x^{-1} \rightarrow -7x^{-2}$
$\frac{dy}{dx} = 15x^2 - 7x^{-2}$
**Answer: $\frac{dy}{dx} = 15x^2 - \frac{7}{x^2}$**

## 4 Averages with algebra

Here are five numbers: $2x - 2$, $x + 5$, $x + 6$, 2, $2x + 3$.
The mean of the five numbers is 10.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(2x - 2) + (x + 5) + (x + 6) + (2) + (2x + 3) = 10 \times 5 = 50$
$6x + 14 = 50$, so $6x = 36$ and $x = 6$
The numbers are 10, 11, 12, 2, 15
(b) In order: 2, 10, 11, 12, 15, so the median is 11
**Answer: (a) x = 6 (b) 11**

## 5 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 8^2 + 4^2 = 80$
$AG^2 = AC^2 + CG^2 = 80 + 1^2 = 81$
AG = 9 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{1}{\sqrt{80}}$

angle GAC = 6.4° (1 d.p.)
**Answer: (a) 9 cm (b) 6.4°**

## 6 Show that dy/dx = ...

$y = \frac{(x + 5)^2}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{(x - 5)(x + 5)}{x^2}$.

(b) Find the gradient when x = 2.

Expand: $(x + 5)^2 = x^2 + 10x + 25$, so $y = x + 10 + 25x^{-1}$

$\frac{dy}{dx} = 1 - \frac{25}{x^2} = \frac{x^2 - 25}{x^2} = \frac{(x - 5)(x + 5)}{x^2}$

When x = 2: $\frac{dy}{dx} = -\frac{21}{4}$

**Answer: (b) $-\frac{21}{4}$**

## 7 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the adjacent side AB, so use cos 60°

$\cos 60° = \frac{1}{2}$

$x = 6 \div \cos 60° = 6 \div \frac{1}{2}$

x = 12
**Answer: x = 12**

## 8 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 7 cm, BC = 10 cm and CG = 12 cm.
(a) Work out the length of AG.
(b) Work out the size of angle GAB.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AG^2 = 7^2 + 10^2 + 12^2 = 293$, so AG = 17.12 cm
(b) $BG^2 = 10^2 + 12^2 = 244$. AB is perpendicular to the plane BCGF, so angle ABG = 90°

Triangle ABG is right-angled at B, so $\cos GAB = \frac{AB}{AG} = \frac{7}{17.12}$

Angle GAB = 65.9°
**Answer: (a) 17.1 cm (b) 65.9°**

## 9 Three stationary points of a quartic

$y = 2x^4 - 16x^2 + 3$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 32x = 8x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$, $x = 0$ or $x = 2$

Points: $(-2, -29)$, $(0, 3)$, $(2, -29)$

$\frac{d^2y}{dx^2} = 24x^2 - 32$

$x = -2$: $\frac{d^2y}{dx^2} = 64$; $x = 0$: $\frac{d^2y}{dx^2} = -32$; $x = 2$: $\frac{d^2y}{dx^2} = 64$

So $(-2, -29)$ is a minimum, $(0, 3)$ is a maximum, $(2, -29)$ is a minimum

**Answer: $(-2, -29)$ minimum; $(0, 3)$ maximum; $(2, -29)$ minimum**

## 10 Tangency conditions

The line $y = kx - 3$ is a tangent to the curve $y = x^2 + 5$.

Find the possible values of k.

Give any surds in their simplest form.

Equate: $x^2 + 5 = kx - 3$

Rearrange: $x^2 - kx + 8 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 8 = 0$

$k^2 = 32$

$k = \pm\sqrt{32} = \pm 4\sqrt{2}$

**Answer: $k = \pm 4\sqrt{2}$**