Day 270 **IGCSE 10-a-day: Model Solutions**

1 Rational and irrational numbers

Here are six numbers.

$\sqrt{8} \times \sqrt{2}, \sqrt{49}, \sqrt{3} \times \sqrt{5}, \sqrt{2} \times \sqrt{6}, 0.\dot{4}, (\sqrt{6} + 2)(\sqrt{6} - 2)$

Write down the irrational numbers.

Simplify each number:

$\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$; $\sqrt{49} = 7$; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

$\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$; $0.\dot{4} = \frac{4}{9}$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$

Answer: $\sqrt{3} \times \sqrt{5}$, $\sqrt{2} \times \sqrt{6}$

2 Ratio and algebra

In a bag, the ratio of the number of red counters to the number of blue counters is 6 : 7.

Then 2 red counters are removed and 11 blue counters are removed.

The ratio is now 7 : 6.

How many red counters were there at the start?

Let the numbers at the start be 6k and 7k

$(6k - 2) : (7k - 11) = 7 : 6$, so $6(6k - 2) = 7(7k - 11)$

$36k - 12 = 49k - 77$

$-13k = -65$, so $k = 5$

Number of red counters at the start: $6 \times 5 = 30$

Answer: 30

3 Gradient where the curve crosses the x-axis

A curve has equation $y = (x + 1)(x - 3)(x + 3)$.

Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 + x^2 - 9x - 9$

$\frac{dy}{dx} = 3x^2 + 2x - 9$

The curve crosses the x-axis at $x = -3$, -1 and 3

Gradients: $x = -3$ gives 12, $x = -1$ gives -8, $x = 3$ gives 24

Answer: 12 at $x = -3$, -8 at $x = -1$, 24 at $x = 3$

4 Solving trigonometric equations

Solve $3 \sin x + 1 = 0$ for $0° \leq x \leq 360°$. (Calculator)

Give your answers correct to 1 decimal place.

$\sin x = -\frac{1}{3}$

Related acute angle: $\sin^{-1}(\frac{1}{3}) = 19.5°$

sine is negative in the 3rd and 4th quadrants, so $x = 180° + 19.5°$ and $x = 360° - 19.5°$

$x = 199.5°$ and $x = 340.5°$

Answer: $x = 199.5°$ or $x = 340.5°$

5 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.

(a) Work out the interquartile range for each.

(b) Compare the distributions of A and B.

(a) IQR of A = $55 - 40 = 15$

IQR of B = $90 - 60 = 30$

(b) Class B has the higher median (85 compared with 50), so on average Class B scored higher

Class A has the smaller interquartile range (15 compared with 30), so its marks are more consistent

Answer: (a) A: 15, B: 30 (b) B has the higher median, A has the smaller IQR

6 Tangent meets the axes

The diagram shows the curve $y = -2x^2 + 6x$ and the tangent at P(1, 4), meeting the axes at A and B.
(a) Find an equation of the tangent at P.
(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = -4x + 6$, so at x = 1 the gradient is 2
$y - 4 = 2(x - 1)$, so $y = 2x + 2$
B: x = 0 gives y = 2. A: y = 0 gives x = −1
Area = ½ × 1 × 2 = 1
Answer: (a) y = 2x + 2 (b) 1

7 Sine rule: finding an angle

Angle B is acute.
Work out the size of angle ABC.
Give your answer correct to 1 decimal place. (Calculator)

$\frac{\sin B}{AC} = \frac{\sin A}{BC}$
$\sin B = \frac{5 \times \sin 56°}{9.2}$
$B = \sin^{-1}(0.4506) = 26.8°$
Answer: 26.8°

8 Open box of fixed surface area

An open tray has a square base of side x cm and height h cm. The total outside surface area is 432 cm².
(a) Show that the volume of the box is $V = \frac{1}{4}(432x - x^3)$ cm³.
(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 432$, so $h = \frac{432 - x^2}{4x}$
$V = x^2h = \frac{x(432 - x^2)}{4} = \frac{1}{4}(432x - x^3)$
$\frac{dV}{dx} = -\frac{3}{4}x^2 + 108 = 0$, so $x^2 = 144$ and x = 12
$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -18 < 0$, so a maximum
V = 864
Answer: 864 cm³

9 Iteration: rearranging equations

(a) Show that the equation $x^3 + 3x - 15 = 0$ has a solution between x = 2 and x = 3.
(b) Show that the equation can be rearranged to give $x = \sqrt[3]{15 - 3x}$

(a) Let $f(x) = x^3 + 3x - 15$. $f(2) = 2^3 + 3 \times 2 - 15 = -1$ and $f(3) = 3^3 + 3 \times 3 - 15 = 21$
f(2) is negative and f(3) is positive. There is a change of sign, so there is a solution between 2 and 3
(b) Rearrange $x^3 + 3x - 15 = 0$: $x^3 = 15 - 3x$
Take the cube root of both sides: $x = \sqrt[3]{15 - 3x}$
Answer: Shown

10 Variation: joint and combined

P is directly proportional to r and inversely proportional to the square of t.
When r = 8 and t = 5, P = 3.84.
Find P when r = 10 and t = 4.

$P = k\frac{r}{t^2}$

Substitute: $3.84 = k \times \frac{8}{25}$ so $k = 12$

$P = 12 \times \frac{5}{8} = 7.5$

Answer: P = 7.5