Day 27 **IGCSE 10-a-day: Model Solutions**

1 Tree diagrams and reverse probability

Machine A makes 20% of the bolts in a factory. Machine B makes the rest.
80% of the bolts from machine A are faulty.
40% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.2 × 0.8 + 0.8 × 0.4 = 0.16 + 0.32 = 0.48
(b) P(A and faulty) = 0.2 × 0.8 = 0.16

P(A | faulty) = 0.16 ÷ 0.48 = $\frac{1}{3}$

Answer: (a) 0.48 (b) $\frac{1}{3}$

2 Stationary point of a quartic

$y = -2x^4 - 64x$
Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -8x^3 - 64$
$\frac{dy}{dx} = 0$: $-8x^3 - 64 = 0$, so $x^3 = -8$
$x = -2$ and $y = 96$
$\frac{d^2y}{dx^2} = -24x^2$

At $x = -2$: $\frac{d^2y}{dx^2} = -96$, so a maximum

Answer: $(-2, 96)$, maximum

3 Inverse proportion in context

6 workers can build a wall in 24 days.
After 6 days, 2 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 6 × 24 = 144 worker-days
Work done = 6 × 6 = 36, so work left = 108
Workers left = 6 − 2 = 4
Time = 108 ÷ 4 = 27 days
Answer: 27 days

4 Equation of a transformed graph

$f(x) = x^2 - 3x + 4$
The graph of $y = f(x)$ is transformed. Find the equation of the graph after it is
(a) reflected in the y-axis
(b) stretched parallel to the x-axis with scale factor $\frac{1}{2}$

(a) $y = f(-x)$: replace x by $-x$. So $y = x^2 + 3x + 4$
(b) $y = f(2x)$: replace x by $2x$. So $y = 4x^2 - 6x + 4$
Answer: (a) $y = x^2 + 3x + 4$ (b) $y = 4x^2 - 6x + 4$

5 Exact trigonometric values

Work out the exact value of cos 45° ÷ cos 30°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 45° \div \cos 30° = \frac{\sqrt{2}}{2} \div \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{6}}{3}$

Answer: $\frac{\sqrt{6}}{3}$

6 Simultaneous equations

Solve the simultaneous equations
$x + 4y = 3$
$5x + y = 34$

To match the x terms: (1) × 5 gives $5x + 20y = 15$
Subtract the equations to eliminate x: $19y = -19$
$y = -1$
Substitute $y = -1$ into (1): $x - 4 = 3$
$x = 7$
Answer: $x = 7$, $y = -1$

7 No stationary points

A curve has equation $y = 2x^3 + 6x^2 + 11x + 2$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 + 12x + 11 = 6(x + 1)^2 + 5$

A squared term is never negative, so $\frac{dy}{dx} \geq 5 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 5 (when $x = -1$)
Answer: (b) 5

8 Tree diagrams and reverse conditional probability

Machine A makes 25% of the items and machine B makes the rest.
6% of items from A are faulty and 8% of items from B are faulty.
An item is chosen at random.
(a) Work out the probability that the item is faulty.
(b) The item is faulty. Work out the probability that it came from machine A.

(a) $0.25 \times 0.06 = 0.015$ and $0.75 \times 0.08 = 0.06$
P(faulty) = 0.015 + 0.06 = 0.075
(b) $\frac{0.015}{0.075} = \frac{1}{5}$

Answer: (a) 0.075 (b) $\frac{1}{5}$

9 Increasing for all x

$f(x) = x^3 + kx^2 + 27x + 5$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 27$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 27 < 0$, so $4k^2 < 324$
$k^2 < 81$, so $-9 < k < 9$
Answer: $-9 < k < 9$

10 Percentages with an unknown rate

The price of a ticket is decreased by x%.
The new price is then increased by x%.
The final price is 25% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 25% less, so the multiplier is 0.75: $1 - \frac{x^2}{10000} = 0.75$

$\frac{x^2}{10000} = 0.25$, so $x^2 = 2500$ and $x = 50$

Answer: x = 50