Day 269 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = -6x^2 - \frac{6}{x^2}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -6x^2 - 6x^{-2}$

$-6x^2 \rightarrow -12x$, $-6x^{-2} \rightarrow 12x^{-3}$

$\frac{dy}{dx} = -12x + 12x^{-3}$

Answer: $\mathbf{\frac{dy}{dx} = -12x + \frac{12}{x^3}}$

2 Where a line meets the axes

The line L has equation $4x - 3y = 12$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $4x = 12$, so $x = 3$. A is (3, 0)
At B, $x = 0$: $-3y = 12$, so $y = -4$. B is (0, −4)
(b) $AB^2 = 3^2 + 4^2 = 25$
$AB = \sqrt{25} = 5$
Answer: (a) A(3, 0), B(0, −4) (b) 5

3 Turning point of a quadratic by differentiation

$f(x) = 3x^2 - 18x + 9$
Use differentiation to find the coordinates of the turning point of the graph of $y = f(x)$. State whether it is a maximum or a minimum.

$f'(x) = 6x - 18$
$f'(x) = 0$: $6x - 18 = 0$, so $x = 3$
$f(3) = -18$
$f''(x) = 6$, which is positive, so it is a minimum
Answer: (3, −18), minimum

4 Shaded areas: exact answers

ABCD is a square of side 6 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 6) = $\frac{1}{4} \times \pi \times 6^2 = 9\pi$

The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area = $2 \times 9\pi - 36 = 18\pi - 36$
Answer: $(18\pi - 36)$ cm^2

5 Variation in context

The time T seconds for a pendulum to swing is directly proportional to the square root of its length l metres.
When $l = 25$, $T = 10$s.
Find the value of T when $l = 9$.

$T = k\sqrt{l}$
$10 = k \times \sqrt{25}$ so $k = 2$
$T = 2\sqrt{l}$
$T = 2 \times 3 = 6$
Answer: 6s

6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 45°

$\cos 45° = \frac{\sqrt{2}}{2}$

$x = 11 \times \cos 45° = 11 \times \frac{\sqrt{2}}{2}$

$x = \frac{11\sqrt{2}}{2}$

Answer: x = $\frac{11\sqrt{2}}{2}$

7 Reverse percentages

The value of a house is decreased by 5% in one year.
The next year its value is decreased by 15%.
It is now worth £88,825.
Work out the value of the house before the first change.

Multipliers: 0.95, 0.85
Overall multiplier = 0.95 × 0.85 = 0.8075
Original = final ÷ multiplier = £88,825 ÷ 0.8075
= £110,000
Answer: £110,000

8 Sine rule: finding an angle

Angle B is acute.
Work out the size of angle ABC.
Give your answer correct to 1 decimal place. (Calculator)

$\frac{\sin B}{AC} = \frac{\sin A}{BC}$

$\sin B = \frac{7.1 \times \sin 68°}{15}$

$B = \sin^{-1}(0.4389) = 26.0°$
Answer: 26.0°

9 Pressure

A metal crate in the shape of a cuboid weighs 840 N.
Its smallest face has area 500 cm^2. Its largest face has area 1,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 500 cm^2 = 0.05 m^2 and 1,500 cm^2 = 0.15 m^2
(a) Greatest pressure is on the smallest face: 840 ÷ 0.05 = 16,800 N/m^2
(b) Least pressure is on the largest face: 840 ÷ 0.15 = 5,600 N/m^2
Answer: (a) 16,800 N/m^2 (b) 5,600 N/m^2

10 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + 5x - 3$ at the point P where $x = -2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 5$: at P, gradient = 1 and $y = -9$

Normal: $y = -x - 11$

Intersection: $x^2 + 6x + 8 = 0$

One root is $x = -2$, and the sum of the roots is -6

So $x_Q = -6 - (-2) = -4$

$y = -7$

Answer: $(-4, -7)$