Day 268 **IGCSE 10-a-day: Model Solutions**

1 Capture and recapture

100 beetles in a field are caught, marked and put back.
Later, a second sample of 30 beetles is caught. 10 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 10 out of 30

This should match the proportion marked in the whole population: $\frac{10}{30} = \frac{100}{N}$

N = 100 × 30 ÷ 10 = 300

(b) No beetles are born, die or leave between the samples, and the beetles mix evenly

Answer: (a) 300 (b) The population stays the same between samples

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

67%, $\frac{4}{5}$, 95%, $\frac{14}{17}$

Convert each to a decimal: 67% = 0.67, $\frac{4}{5}$ = 0.8, 95% = 0.95, $\frac{14}{17}$ = 0.824

In order: 67%, $\frac{4}{5}$, $\frac{14}{17}$, 95%

Answer: 67%, $\frac{4}{5}$, $\frac{14}{17}$, 95%

3 Quadratic formula and surds

Solve $2x^2 + 6x + 2 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with a = 2, b = 6, c = 2

$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 2 \times 2}}{2 \times 2} = \frac{-6 \pm \sqrt{20}}{4}$

x = −2.6180... or x = −0.3820...

x = −2.62 or x = −0.38 (2 d.p.)

Answer: x = −2.62 or x = −0.38

4 Tree diagrams and reverse conditional probability

Machine A makes 75% of the items and machine B makes the rest.
1% of items from A are faulty and 6% of items from B are faulty.
An item is chosen at random.
(a) Work out the probability that the item is faulty.
(b) The item is faulty. Work out the probability that it came from machine A.

(a) 0.75 × 0.01 = 0.0075 and 0.25 × 0.06 = 0.015

P(faulty) = 0.0075 + 0.015 = 0.0225

(b) $\frac{0.0075}{0.0225} = \frac{1}{3}$

Answer: (a) 0.0225 (b) $\frac{1}{3}$

5 Stationary points and their nature

$y = -x^3 + 6x^2 - 9x - 4$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = -3x^2 + 12x - 9 = -3(x - 1)(x - 3)$

$\frac{dy}{dx} = 0$ gives $x = 1$ or $x = 3$, so the points are $(1, -8)$ and $(3, -4)$

$\frac{d^2y}{dx^2} = -6x + 12$

$x = 1$: $\frac{d^2y}{dx^2} = 6$, which is positive, so a minimum

$x = 3$: $\frac{d^2y}{dx^2} = -6$, which is negative, so a maximum

Answer: (a) (1, −8) and (3, −4) (b) (1, −8) is a minimum, (3, −4) is a maximum

6 Pythagoras with algebra

ABC is a right-angled triangle with sides $(x - 2)$ cm, (x) cm and $(x + 2)$ cm. The longest side is $(x + 2)$ cm.

(a) Find the value of x.

(b) Work out the area of the triangle.

The diagram is not drawn accurately.

(a) Pythagoras: $(x - 2)^2 + x^2 = (x + 2)^2$

Expand and collect terms: $x^2 - 8x = 0$

Factorise: $x(x - 8) = 0$

$x = 8$ or $x = 0$, and x must be positive, so $x = 8$

(b) The two shorter sides are 6 cm and 8 cm

Area $= \frac{1}{2} \times 6 \times 8 = 24$ cm^2

Answer: (a) x = 8 (b) 24 cm^2

7 Curves that touch

The curves $y = -x^2 + 2x + 3$ and $y = -3x^2 - 2x + 1$ touch at one point.

Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $2x^2 + 4x + 2 = 0$

That is $2(x + 1)^2 = 0$ with a repeated root $x = -1$

$y = 0$, so the point is $(-1, 0)$

$\frac{dy}{dx} = -2x + 2$ on the first curve: gradient 4

Tangent: $y = 4x + 4$

Answer: (−1, 0), y = 4x + 4

8 Exact lengths and areas

A rectangle has area $(26 + 14\sqrt{3})$ cm^2 and length $(2 + 2\sqrt{3})$ cm.

Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{3}$

Width = area ÷ length $= \frac{26 + 14\sqrt{3}}{2 + 2\sqrt{3}}$

Multiply top and bottom by $(2 - 2\sqrt{3})$:

Numerator: $(26 + 14\sqrt{3})(2 - 2\sqrt{3}) = -32 - 24\sqrt{3}$

Denominator: $4 - 12 = -8$

Width $= 4 + 3\sqrt{3}$

Answer: $4 + 3\sqrt{3}$ cm

9 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 - 2x$ at the point P where x = 2 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x - 2$: at P, gradient = 2 and y = 0

Normal: $y = -\frac{1}{2}x + 1$

Intersection: $x^2 - \frac{3}{2}x - 1 = 0$

One root is x = 2, and the sum of the roots is $\frac{3}{2}$

So $x_Q = \frac{3}{2} - 2 = -\frac{1}{2}$

$y = \frac{5}{4}$

Answer: $\left(-\frac{1}{2}, \frac{5}{4}\right)$

10 Bounds of a compound expression

p = 11.5 and r = 7.9, each correct to 1 decimal place.
q = 2.21, correct to 2 decimal places.

Work out the upper bound of $\frac{p+q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom
Top: UB p + UB q = 11.55 + 2.215 = 13.765
Bottom: LB r = 7.85
13.765 ÷ 7.85 = 1.75
Answer: 1.75