Day 267 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 30)°$, $(x + 40)°$, $(x + 15)°$, $(x + 5)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$5x + 90 = 540$, so $5x = 450$
$x = 90$
(b) The angles are 90°, 120°, 130°, 105°, 95°
The largest is 130°
Answer: (a) x = 90 (b) 130°

2 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = 4x^3 + 6x^2 - 7$

Differentiate each term: $x^n \rightarrow nx^{n-1}$ and a constant $\rightarrow 0$
$4x^3 \rightarrow 12x^2$, $6x^2 \rightarrow 12x$
$\frac{dy}{dx} = 12x^2 + 12x$
Answer: $\mathbf{\frac{dy}{dx} = 12x^2 + 12x}$

3 Describe a graph transformation

Describe fully the single transformation that maps the graph of $y = f(x)$ onto the graph of
(a) $y = f(x) - 5$
(b) $y = f(-x)$

(a) Subtracting 5 from f(x) moves the graph down 5 units
(b) Changing the sign of every x value flips the graph over the y-axis
Answer: (a) A translation by the vector $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ **(b) A reflection in the y-axis**

4 Equation of a tangent

A curve has equation $y = 2x^3 - 6x - 2$.
Find the equation of the tangent to the curve at the point where $x = -2$.

When $x = -2$, $y = -6$
$\frac{dy}{dx} = 6x^2 - 6$, so the gradient at $x = -2$ is 18
$y + 6 = 18(x + 2)$
$y = 18x + 30$
Answer: y = 18x + 30

5 Bounds of area and perimeter

A rectangle has length 11 cm and width 15 cm.
Both measurements are correct to the nearest centimetre.
Tom says that the area of the rectangle could be exactly 151 cm^2.
Is Tom correct? You must show your working.

$10.5 \leq$ length < 11.5 and $14.5 \leq$ width < 15.5
Lower bound of area $= 10.5 \times 14.5 = 152.25$
Upper bound of area $= 11.5 \times 15.5 = 178.25$
So $152.25 \leq$ area < 178.25
151 is outside this interval, so Tom is not correct
Answer: No, 151 is outside the interval

6 Fencing against a wall

A farmer uses 112 m of fencing to enclose a rectangle against a wall, as shown.

(a) Show that the area, A m^2, is $A = 112x - 2x^2$.

(b) Find the maximum area, proving it is a maximum.

Side parallel to the wall = $112 - 2x$, so $A = x(112 - 2x) = -2x^2 + 112x$

$\frac{dA}{dx} = -4x + 112 = 0$ gives $x = 28$

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

$A = 112 \times 28 - 2 \times 28^2 = 1568$

Answer: 1568 m^2

7 Conditional probability from a table

The table shows information about 103 children.

(a) A child who chooses tennis is chosen at random. Work out the probability that this child is a girl.

(b) Two of the 103 children are chosen at random. Work out the probability that both are girls.

(a) 45 children are in the Tennis column, and 18 of them are girls

Probability = $\frac{2}{5}$

(b) There are 59 girls, so P(both) = $\frac{59}{103} \times \frac{58}{102} = \frac{1711}{5253}$

Answer: (a) $\frac{2}{5}$ (b) $\frac{1711}{5253}$

8 Tree diagrams and reverse conditional probability

30% of a town's people have been vaccinated.

9% of vaccinated people catch a virus and 3% of unvaccinated people catch it.

A person is chosen at random.

(a) Work out the probability that the person catches the virus.

(b) The person has caught the virus. Work out the probability that they were vaccinated.

(a) $0.3 \times 0.09 = 0.027$ and $0.7 \times 0.03 = 0.021$

P(faulty) = 0.027 + 0.021 = 0.048

(b) $\frac{0.027}{0.048} = \frac{9}{16}$

Answer: (a) 0.048 (b) $\frac{9}{16}$

9 Probability and quadratic equations

A bag contains some counters. 8 of them are green and the rest are white.

Two counters are taken at random without replacement.

The probability that both are green is $\frac{28}{45}$.

Work out how many counters are white.

Let there be n counters altogether

P(both green) = $\frac{8}{n} \times \frac{7}{n-1} = \frac{56}{n(n-1)}$

$\frac{56}{n(n-1)} = \frac{28}{45}$, so $n(n-1) = 90$, giving $n^2 - n - 90 = 0$

$(n - 10)(n + 9) = 0$, so $n = 10$ (n cannot be negative)

Number of counters that are white = $10 - 8 = 2$

Answer: 2

10 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = x^2 + 3x - 3$.
Find the value of k.

Equate: $x^2 + 3x - 3 = -2x + k$

Rearrange: $x^2 + 5x - 3 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $25 - 4 \times 1 \times (-3 - k) = 0$

$4k + 37 = 0$

$k = -\frac{37}{4}$

Answer: $k = -\frac{37}{4}$