Day 266 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:45 (London time).
It takes 9 hours 25 minutes.
When it is 12:00 in London it is 17:00 in Delhi.
At what time, in Delhi time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:45 + 9 h 25 min = 16:10
Delhi is 5 hours ahead of London, so add 5 hours
21:10
Answer: 21:10 the same day

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = -t^3 + 2t^2 - 9t$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = -3t^2 + 4t - 9$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -6t + 4$

When t = 3: $v = -24$ and $a = -14$
Answer: (a) -24 m/s (b) -14 m/s^2

3 Inclusion and exclusion with two sets

There are 63 employees.
11 of them speak German and 15 of them speak Italian.
44 of them do neither.
(a) Work out how many do both.
(b) One of the employees is chosen at random. Find the probability that they only speak German.

Number in A or B = 63 − 44 = 19
(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 11 + 15 - 19 = 7$
(b) Number who only speak German = 11 − 7 = 4

Probability = $\frac{4}{63}$

Answer: (a) 7 (b) $\frac{4}{63}$

4 Pressure

A concrete slab in the shape of a cuboid weighs 720 N.
Its smallest face has area 1000 cm^2. Its largest face has area 3,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 1000 cm^2 = 0.1 m^2 and 3,000 cm^2 = 0.3 m^2
(a) Greatest pressure is on the smallest face: 720 ÷ 0.1 = 7,200 N/m^2
(b) Least pressure is on the largest face: 720 ÷ 0.3 = 2,400 N/m^2
Answer: (a) 7,200 N/m^2 (b) 2,400 N/m^2

5 Increasing and decreasing with powers

f(x) = $x^{\frac{3}{2}} - 3x$ for x > 0

(a) Find the range of values of x for which f is a decreasing function.

(b) Find the least value of f(x).

f′(x) = $\frac{3}{2}x^{\frac{1}{2}} - 3$

f′(x) < 0 when $\frac{3}{2}x^{\frac{1}{2}} < 3$, so $x^{\frac{1}{2}} < 2$

Square both sides: x < 4, so 0 < x < 4

f decreases then increases, so the least value is at x = 4: f = $4^{\frac{3}{2}} - 3 \times 4 = 8 - 12 = -4$

Answer: (a) 0 < x < 4 (b) −4

6 Growth and decay

A machine is worth £1,000 when new.

Its value decreases by 8% each year.

After how many whole years will it first be worth less than £750? (Calculator)

Multiplier each year = 0.92

After 3 years: £1,000 × 0.92^3 = £778.69

After 4 years: £1,000 × 0.92^4 = £716.39

So the first time it is less than £750 is after 4 years

Answer: 4 years

7 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 4n - 4$.

(a) Work out the 8th term.

(b) Is 239 a term of the sequence? You must show your working.

(a) $2 \times 8^2 + 4 \times 8 - 4 = 156$

(b) Solve $2n^2 + 4n - 4 = 239$, so $2n^2 + 4n - 243 = 0$

$b^2 - 4ac = 4^2 - 4 \times 2 \times (-243) = 1960$, which is not a square number

So n is not a whole number, and 239 is not a term

Answer: (a) 156 (b) No

8 No stationary points

A curve has equation $y = 2x^3 + 6x^2 + 14x - 4$.

(a) Show that the curve has no stationary points.

(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 + 12x + 14 = 6(x + 1)^2 + 8$

A squared term is never negative, so $\frac{dy}{dx} \geq 8 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 8 (when x = −1)

Answer: (b) 8

9 Loci and regions

ABCD is a rectangle with AB = 12 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 10^2 = \frac{25}{2}\pi$ cm^2

Answer: $\frac{25}{2}\pi$ cm^2

10 Range and inverse of a quadratic

$g(x) = x^2 - 12x + 34$ for $x \leq 6$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x - 6)^2 - 2$
$x \leq 6$ includes the minimum point, so $g(x) \geq -2$
(b) $y = (x - 6)^2 - 2$, so $(x - 6)^2 = y + 2$
$x \leq 6$ means $x - 6$ is negative or zero, so $x = 6 - \sqrt{y + 2}$
$g^{-1}(x) = 6 - \sqrt{x + 2}$

Answer: (a) $g(x) \geq -2$ (b) $g^{-1}(x) = 6 - \sqrt{x + 2}$