Day 265 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 20 is taken. Work out how many come from Group D.
(b) A different stratified sample has 6 from Group C. Work out the size of this sample.

Total = 350 + 250 + 150 + 250 = 1000

(a) Group D: $\frac{250}{1000} \times 20 = 5$

(b) Fraction sampled = 6 ÷ 150 = $\frac{1}{25}$, so sample size = $\frac{1}{25} \times 1000 = 40$

Answer: (a) 5 (b) 40

2 Find unknown constants in a function

$f(x) = ax^2 + b$, where a and b are constants.
$f(1) = 6$ and $f(6) = -64$.
Find the value of a and the value of b.

$a + b = 6$ and $36a + b = -64$
Subtract: $35a = -70$, so $a = -2$
$b = 6 - (-2) \times 1 = 8$
Answer: $a = -2$, $b = 8$

3 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = 2t^3 + 8t^2 + 7t + 4$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = 6t^2 + 16t + 7$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t + 16$

When t = 3: v = 109 and a = 52
Answer: (a) 109 m/s (b) 52 m/s^2

4 Tests of divisibility

The 4-digit number 78k3 is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 7 + 8 + 3 + k = 18 + k
This must be a multiple of 3, so k = 0 or 3 or 6 or 9
Answer: k = 0 or 3 or 6 or 9

5 Where is the second derivative zero?

$y = 3x^3 - 27x^2 + 6x + 7$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = 9x^2 - 54x + 6$

$\frac{d^2y}{dx^2} = 18x - 54$

$\frac{d^2y}{dx^2} = 0$ gives $18x - 54 = 0$, so $x = 3$

$\frac{dy}{dx} = -75$ when $x = 3$

Answer: (a) $18x - 54$ (b) $x = 3$, gradient -75

6 Enlargement

Triangle T has vertices A(2, 3), B(0, 0), C(3, 0).
T is enlarged with scale factor −2 and centre (3, 2).
(a) Write down the coordinates of the image of A.
(b) The area of T is 18 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (2 − 3, 3 − 2) = (−1, 1)
Multiply by −2: (2, −2)
Image of A = centre + this vector = (5, 0)

(b) Area scale factor = $(-2)^2 = 4$
Area = 18 × 4 = 72 square units
Answer: (a) (5, 0) (b) 72

7 Rearranging when the subject appears twice

Make t the subject of T(t + 4) = t − T

Expand the bracket: Tt + 4T = t − T
Collect the t terms: Tt − t = −4T − T
Factorise: t(T − 1) = −5T

$t = \frac{5T}{1 - T}$

Answer: $\mathbf{t = \frac{5T}{1 - T}}$

8 Bearings with sine and cosine rules

A ship sails 14 km from P to Q on a bearing of 037°.
It then sails 15 km from Q to R on a bearing of 156°.
(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 037° + 180° − 156° = 61°

$PR^2 = 14^2 + 15^2 - 2 \times 14 \times 15 \times \cos 61°$ so PR = 14.7 km

$\cos QPR = \frac{14^2 + 14.744^2 - 15^2}{2 \times 14 \times 14.744}$, angle QPR = 62.85°

Bearing of R from P = 037° + 62.85° = 100°
Answer: (a) 14.7 km (b) 100°

9 Increasing range for y = ax + b/x

$y = 2x + \frac{32}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 2 - \frac{32}{x^2}$

$\frac{dy}{dx} > 0$: $2 > \frac{32}{x^2}$

$x^2 > 16$ (x^2 is positive, so multiply through)
x < −4 or x > 4
Answer: x < −4 or x > 4

10 Compound interest

Dina invests £10,000 for 7 years at 2.5% per year compound interest.
Work out the value of the investment at the end of the 7 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 2.5 ÷ 100 = 1.025

$10000 \times 1.025^7 = 11886.86$

To the nearest penny: £11,886.86
Answer: £11,886.86