Day 264 **IGCSE 10-a-day: Model Solutions**

1 Error intervals

The capacity of a tank is 670 litres, correct to the nearest 10.
Write down the error interval for the capacity of a tank, using c.

The value has been rounded to the nearest 10
Half of this is 5, so go 5 below and 5 above
670 − 5 = 665 and 670 + 5 = 675
$665 \leq c < 675$
Answer: $665 \leq c < 675$

2 Determinant and inverse

$M = \begin{pmatrix} 0 & 3 \\ 2 & 7 \end{pmatrix}$.
(a) Work out the determinant of M.
(b) Find M^{-1}.

(a) det M = 0 × 7 − 3 × 2 = −6
(b) Swap the leading diagonal, change the signs of the other two entries, divide by the determinant
$M^{-1} = -\frac{1}{6}\begin{pmatrix} 7 & -3 \\ -2 & 0 \end{pmatrix}$
Answer: (a) -6 (b) $-\frac{1}{6}\begin{pmatrix} 7 & -3 \\ -2 & 0 \end{pmatrix}$

3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 1 : 2.
80% of the Year 10 students and 20% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 60 students in total: 20 Year 10 students and 40 Year 11 students
80% of 20 = 16 and 20% of 40 = 8
Together: 24 out of 60
24 ÷ 60 × 100 = 40%
Answer: 40%

4 Nature from the gradient either side

$y = -2x^3 + 6x + 1$
(a) Show that there is a stationary point at x = 1.
(b) By finding the gradient when $x = \frac{1}{2}$ and when $x = \frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 + 6$, so when x = 1: $\frac{dy}{dx} = 0$
$x = \frac{1}{2}$: gradient = $\frac{9}{2}$ (positive)
$x = \frac{3}{2}$: gradient = $-\frac{15}{2}$ (negative)
The gradient goes from positive to 0 to negative, so (1, 5) is a maximum
Answer: (b) maximum

5 Simple and compound interest

Ivan invests £3,000 for 3 years at 3.5% per year simple interest.
Chen invests £3,000 for 3 years at 3.5% per year compound interest.
Work out how much more interest Chen earns than Ivan.
Give your answer to the nearest penny. (Calculator)

Simple interest: £3,000 × 3.5% × 3 = £315
Compound: £3,000 × 1.035^3 = £3,326.15
Compound interest = £3,326.15 − £3,000 = £326.15
Difference = £326.15 − £315 = £11.15
Answer: £11.15

6 Show that a line is a tangent

The line $y = -5x - 2$ meets the curve $y = x^2 + x + 7$.
Show that the line is a tangent to the curve and find the coordinates of the point where they touch.

Equate: $x^2 + x + 7 = -5x - 2$, so $x^2 + 6x + 9 = 0$

Discriminant: $6^2 - 4 \times 1 \times 9 = 36 - 36 = 0$

One repeated root, so the line touches the curve at one point only: it is a tangent

$x = \frac{-6}{2 \times 1} = -3$ and $y = 13$

Answer: $(-3, 13)$

7 Angles of elevation from two points

Points A and B are on level ground, 86 m apart, with B between A and the foot of a cliff.
The angle of elevation of the top of the cliff is 29° from A and 44° from B.
Work out the height of the cliff. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 44° − 29° = 15° (exterior angle of the triangle)

Sine rule in triangle ABT: $\frac{AT}{\sin 136°} = \frac{86}{\sin 15°}$

$AT = \frac{86 \times \sin 44°}{\sin 15°} = 230.82$

Height = AT × sin 29° = 112 m

Answer: 112 m

8 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $4\mathbf{a} + 5\mathbf{b}$, OQ = $\mathbf{a} - \mathbf{b}$ and OR = $-3\mathbf{a} - 9\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-3\mathbf{a} - 6\mathbf{b}$
QR = OR − OQ = $-4\mathbf{a} - 8\mathbf{b}$

QR = $\frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4

Answer: (b) 3 : 4

9 Composite functions: fg and gf

$f(x) = -3x + 1$ and $g(x) = x^2$
Solve fg(x) = gf(x)

$fg(x) = f(x^2) = -3x^2 + 1$

$gf(x) = g(-3x + 1) = (-3x + 1)^2 = 9x^2 - 6x + 1$

Set them equal and rearrange: $12x^2 - 6x = 0$

Divide by 6 and factorise: $x(2x - 1) = 0$

$x = 0$ or $x = \frac{1}{2}$

Answer: $x = 0$ or $x = \frac{1}{2}$

10 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + x + 2$ at the point P where $x = 0$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 1$: at P, gradient = 1 and $y = 2$

Normal: $y = -x + 2$

Intersection: $x^2 + 2x = 0$

One root is $x = 0$, and the sum of the roots is -2

So $x_Q = -2 - 0 = -2$

$y = 4$

Answer: $(-2, 4)$