Day 263 **IGCSE 10-a-day: Model Solutions**

1 Standard form problems

A virus has a diameter of 1.25×10^{-9} metres.
Viruses are placed in a straight line, touching, to make a length of 1.225×10^{-3} metres.
How many viruses are in the line?
Give your answer in standard form.

Number = total length ÷ diameter
$(1.225 \times 10^{-3}) \div (1.25 \times 10^{-9})$
$1.225 \div 1.25 = 0.98$ and $10^{-3} \div 10^{-9} = 10^{6}$
$= 9.8 \times 10^{5}$
Answer: 9.8×10^{5}

2 Gradient with fractional or negative powers

The equation of a curve is $y = -2x^2 - \frac{8}{x}$.
Work out the gradient of the curve at the point where $x = 4$.

$\frac{dy}{dx} = -4x + 8x^{-2}$

When $x = 4$: gradient $= -\frac{31}{2}$

Answer: $-\frac{31}{2}$

3 Completing the square

(a) Write $2x^2 - 4x + 8$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 2x^2 - 4x + 8$

(a) Take out the factor 2 from the x terms: $2(x^2 - 2x) + 8$
$x^2 - 2x = (x - 1)^2 - 1$, so $2x^2 - 4x + 8 = 2[(x - 1)^2 - 1] + 8$
$= 2(x - 1)^2 + 6$
(b) The minimum is when $x - 1 = 0$, so $x = 1$ and $y = 6$
Answer: (a) $2(x - 1)^2 + 6$ (b) (1, 6)

4 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the hypotenuse AC, so use cos 30°

$\cos 30° = \frac{\sqrt{3}}{2}$

$x = 5 \times \cos 30° = 5 \times \frac{\sqrt{3}}{2}$

$x = \frac{5\sqrt{3}}{2}$

Answer: $x = \frac{5\sqrt{3}}{2}$

5 Singular matrices

The matrix $\begin{pmatrix} k & 2 \\ -1 & 2 \end{pmatrix}$ has no inverse.
Find the value of k.

No inverse means the determinant is 0
$2k + 2 = 0$
$2k = -2$ so $k = -1$
Answer: $k = -1$

6 Standard form

Work out $(4.6 \times 10^{-3}) + (1.5 \times 10^{-6})$
Give your answer in standard form.

Write the first number as a multiple of 10^{-6}: $4.6 \times 10^{-3} = 4600 \times 10^{-6}$

$(4600 + 1.5) \times 10^{-6} = 4601.5 \times 10^{-6}$

$= 4.6015 \times 10^{-3}$

Answer: 4.6015×10^{-3}

7 Probability with sets

$n(\xi) = 80$, $n(A) = 33$, $n(B) = 21$ and $n(A \cap B) = 11$.
An element of ξ is chosen at random.
(a) Find the probability that it is in $A \cup B$.
(b) Given that it is in B, find the probability that it is also in A.

$n(A \cup B) = 33 + 21 - 11 = 43$

(a) $P(A \cup B) = \frac{43}{80}$

(b) Only the 21 elements in B can be chosen, and 11 of these are in A

$P = \frac{11}{21}$

Answer: (a) $\frac{43}{80}$ (b) $\frac{11}{21}$

8 Stationary point with a 1/x term

A curve has equation $y = 3x^2 - \frac{6}{x}$.
(a) Show that the curve has a stationary point at $x = -1$.
(b) Find the y coordinate of this point and decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x + \frac{6}{x^2}$

When $x = -1$: $\frac{dy}{dx} = -6 - (-6) = 0$

$y = 9$

$\frac{d^2y}{dx^2} = 6 - \frac{12}{x^3}$

At $x = -1$: $\frac{d^2y}{dx^2} = 18$, which is positive, so a minimum

Answer: $y = 9$, minimum

9 Winning a match

Rhys and Beth play a match. The first to win 2 chess games wins the match.
The probability that Rhys wins any one of the chess games is $\frac{1}{3}$.
Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Rhys wins then loses) $= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

P(Rhys loses then wins) $= \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Total $= 2 \times \frac{2}{9} = \frac{4}{9}$

Answer: $\frac{4}{9}$

10 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 + x$ at the point P where $x = 0$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x + 1$: at P, gradient = 1 and $y = 0$

Normal: $y = -x$

Intersection: $x^2 + 2x = 0$

One root is $x = 0$, and the sum of the roots is -2

So $x_Q = -2 - 0 = -2$

$y = 2$

Answer: $(-2, 2)$