Day 262 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 17 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 17e.
Interior angle + exterior angle = 180°, so 17e + e = 180°
e = 180° ÷ 18 = 10°
Number of sides = 360° ÷ 10° = 36
Answer: 36

2 Simultaneous equations

Solve the simultaneous equations
$-3x + y = 1$
$3x + 5y = -13$

To match the y terms: (1) × 5 gives $-15x + 5y = 5$
Subtract the equations to eliminate y: $-18x = 18$
$x = -1$
Substitute $x = -1$ into (1): $y + 3 = 1$
$y = -2$
Answer: $x = -1$, $y = -2$

3 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 3. N is on AB with AN : NB = 3 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{3}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a}) = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

(b) OM = $\frac{1}{4}\mathbf{a}$

MN = ON − OM = $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} - \frac{1}{4}\mathbf{a} = \frac{3}{20}\mathbf{a} + \frac{3}{5}\mathbf{b}$

Answer: (a) $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ (b) $\frac{3}{20}\mathbf{a} + \frac{3}{5}\mathbf{b}$

4 Turning point of a quadratic by differentiation

$f(x) = 2x^2 - 12x - 6$
Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = 4x - 12$
$f'(x) = 0$: $4x - 12 = 0$, so $x = 3$
$f(3) = -24$
$f''(x) = 4$, which is positive, so it is a minimum
Answer: (3, −24), minimum

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5 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.

$u_1 = 3$ and $u_2 = x$.

(a) Show that $u_5 = 3x + 6$.

(b) Given that $u_5 = 24$, find x and hence work out u_8.

(a) $u_3 = 3 + x$, $u_4 = u_3 + u_2 = 2x + 3$, $u_5 = u_4 + u_3 = 3x + 6$

(b) $3x + 6 = 24$ so $3x = 18$ and $x = 6$

Terms: 3, 6, 9, 15, 24, 39, 63, 102

$u_8 = 102$

Answer: $x = 6$, $u_8 = 102$

6 Percentage change in area and volume

The edge length of a cube is increased by 50%.

Work out the percentage change in the volume of the cube.

Side length multiplier = 1.5

Volume multiplier = $1.5^3 = 3.375$

$3.375 - 1 = 2.375$, so a 237.5% increase

Answer: 237.5% increase

7 Bounds: finding an unknown length

A rectangle has area 52 cm^2, correct to 2 significant figures.

Its length is 7.900000000000000355271367880050092935562133789062 5 cm, correct to 2 significant figures.

Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: $51.5 \leq A < 52.5$. Length: $7.850000000000000355271367880050092935562133789062 5 \leq l < 7.950000000000000355271367880050092935562133789062 5$

UB width = UB area ÷ LB length = 52.5 ÷ 7.850000000000000355271367880050092935562133789062 5 = 6.69

Answer: 6.69 cm

8 Stationary points of a quartic

A curve has equation $y = x^4 - 2x^2 + 2$.

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 4x = 4x(x - 1)(x + 1)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 1$ or $x = -1$

$y = 2$ when $x = 0$, and $y = 1$ when $x = \pm 1$

$\frac{d^2y}{dx^2} = 12x^2 - 4$: it is -4 at $x = 0$ (maximum) and 8 at $x = \pm 1$ (minimum)

Answer: (0, 2) maximum; (1, 1) and (−1, 1) minima

9 Reverse percentage growth and decay

The population of a town grows by 20% each year.

After 2 years the population is 18,000.

Work out the population 2 years ago. (Calculator)

Yearly multiplier = 1.2

Original $\times 1.2^2 = 18{,}000$

Original $= 18{,}000 \div 1.2^2 = 12{,}500$

Answer: 12,500

10 Normal meets the curve again (quadratic)

The normal to the curve $y = -x^2 - 3x + 4$ at the point P where $x = -2$ meets the curve again at Q. Find the coordinates of Q.

$\frac{dy}{dx} = -2x - 3$: at P, gradient = 1 and $y = 6$

Normal: $y = -x + 4$

Intersection: $-x^2 - 2x = 0$

One root is $x = -2$, and the sum of the roots is -2

So $x_Q = -2 - (-2) = 0$

$y = 4$

Answer: (0, 4)