Day 261 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 23 is taken. Work out how many come from Group A.
(b) A different stratified sample has 9 from Group D. Work out the size of this sample.

Total = 200 + 140 + 60 + 60 = 460

(a) Group A: $\frac{200}{460} \times 23 = 10$

(b) Fraction sampled = $9 \div 60 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 460 = 69$

Answer: (a) 10 (b) 69

2 f′(x) notation

$f(x) = -x^2 - 5x + 4$
(a) Find f′(x).
(b) Work out the value of f′(1).

(a) $-x^2 \to -2x$, $-5x \to -5$
$f'(x) = -2x - 5$
(b) $f'(1) = -7$
Answer: (a) $f'(x) = -2x - 5$ (b) -7

3 Simplifying surds

Write $\sqrt{8} - \sqrt{2} + \sqrt{72}$ in the form $a\sqrt{2}$
where a is an integer.

$\sqrt{8} = 2\sqrt{2}$; $\sqrt{72} = 6\sqrt{2}$
$(2 - 1 + 6)\sqrt{2} = 7\sqrt{2}$
Answer: $7\sqrt{2}$

4 Equation of a circle

A circle has centre C(2, −4) and passes through the point P(3, 0).
Find an equation of the circle.

$r^2 = CP^2 = (3 - 2)^2 + (0 - (-4))^2$
$= 1 + 16 = 17$
$(x - 2)^2 + (y + 4)^2 = 17$
Answer: $(x - 2)^2 + (y + 4)^2 = 17$

5 Doubling and repeated growth

The number of a virus sample triples every 30 hours. At the start there are 800.
How many are there after 90 hours?

90 ÷ 30 = 3 lots of 30 hours
Number = 800×3^3
= 21,600
Answer: 21,600

6 Fractional power functions

$f(x) = 6x^{1/2} - 2x^{-1/2}$
Find f′(x) and work out the value of f′(4).

$6x^{1/2} \to 3x^{-1/2}$, $-2x^{-1/2} \to x^{-3/2}$
$f'(x) = 3x^{-1/2} + x^{-3/2}$
f′(4): $\sqrt{4} = 2$, so $f'(4) = \frac{13}{8}$
Answer: $f'(x) = 3x^{-1/2} + x^{-3/2}$, $f'(4) = \frac{13}{8}$

7 Probability with sets

$n(\xi) = 40$, $n(A) = 8$, $n(B) = 19$ and $n(A \cap B) = 2$.
An element of ξ is chosen at random.
(a) Find the probability that it is in neither A nor B.
(b) Given that it is in A, find the probability that it is not in B.

$n(A \cup B) = 8 + 19 - 2 = 25$

(a) $n(A \cup B)' = 40 - 25 = 15$, so $P = \frac{15}{40} = \frac{3}{8}$

(b) In A there are 8 elements, and $8 - 2 = 6$ of them are not in B

$P = \frac{6}{8} = \frac{3}{4}$

Answer: (a) $\frac{3}{8}$ (b) $\frac{3}{4}$

8 Rationalising the denominator

Write $\frac{5+\sqrt{7}}{3-\sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are rational numbers.

Multiply top and bottom by $(3 + \sqrt{7})$:
Numerator: $(5 + \sqrt{7})(3 + \sqrt{7}) = 22 + 8\sqrt{7}$
Denominator: $9 - 7 = 2$
$= 11 + 4\sqrt{7}$

Answer: $11 + 4\sqrt{7}$

9 Loci and regions

ABCD is a rectangle with AB = 8 cm and AD = 12 cm.
A point P inside the rectangle is such that $AP \leq 8$ cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 4 cm from A
$AP \leq 8$ cm: P is inside the quarter circle of radius 8 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{4}{8}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 4 cm, height $8 \sin 60° = 4\sqrt{3}$ cm, area $= \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 8^2 = \frac{16}{3}\pi$

Area $= 8\sqrt{3} + \frac{16}{3}\pi$ cm^2

Answer: $(8\sqrt{3} + \frac{16}{3}\pi)$ cm^2

10 Tangent to a curve with negative or fractional powers

A curve has equation $y = -x^2 - \frac{4}{x}$.

Find the equation of the tangent to the curve at the point where x = 1.

$\frac{dy}{dx} = -2x + 4x^{-2}$
At x = 1: y = −5 and gradient = 2
$y + 5 = 2(x - 1)$
$y = 2x - 7$

Answer: $y = 2x - 7$