Day 260 **IGCSE 10-a-day: Model Solutions**

1 Show that dy/dx factorises

$y = (x + 4)^2(x + 3)$

Show that $\frac{dy}{dx} = (x + 4)(3x + 10)$.

Expand: $y = x^3 + 11x^2 + 40x + 48$

$\frac{dy}{dx} = 3x^2 + 22x + 40$

Factorise: $3x^2 + 22x + 40 = (x + 4)(3x + 10)$

Answer: $\frac{dy}{dx} = (x + 4)(3x + 10)$

2 Factorising fully

Factorise fully $72x^2 - 18y^2$

Take out the common factor 2: $2(36x^2 - 9y^2)$

$36x^2 - 9y^2$ is a difference of two squares: $(6x)^2 - (3y)^2$

$= (6x - 3y)(6x + 3y)$

Answer: $2(6x - 3y)(6x + 3y)$

3 Exact trigonometric values

Work out the exact value of cos 60° + cos 45°.

Give your answer in its simplest form, with no decimals.

$\cos 60° = \frac{1}{2}$ and $\cos 45° = \frac{\sqrt{2}}{2}$

Substitute and simplify (rationalise any denominator)

$= \frac{1 + \sqrt{2}}{2}$

Answer: $\frac{1 + \sqrt{2}}{2}$

4 Trigonometric graphs

The diagram shows the graph of $y = a \sin bx$, where a and b are positive integers.

(a) Write down the value of a and the value of b.

(b) Work out the value of y when x = 290°.

(a) The maximum value is 4, so a = 4

The graph completes 3 full cycles between 0° and 360°, so b = 3

(b) $y = 4 \sin (3 \times 290°) = 4 \sin 870°$

$\sin 150° = \frac{1}{2}$, so y = 2

Answer: (a) a = 4, b = 3 (b) y = 2

5 Increasing for x greater than k

The function $f(x) = 3x^2 - 18x + 2$ is increasing when x > k.

Find the value of k.

$f'(x) = 6x - 18$

The turning point is where $f'(x) = 0$: $6x - 18 = 0$

$x = 3$

$f'(x)$ is positive for x > 3, so k = 3

Answer: k = 3

6 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 8^2 + 8^2 = 128$, so $AM^2 = (AC \div 2)^2 = 32$
$VM^2 = VA^2 - AM^2 = 6^2 - 32 = 4$
VM = 2 cm

(b) Volume = $\frac{1}{3} \times 8^2 \times 2 = \frac{128}{3}$ cm^3

Answer: (a) 2 cm (b) $\frac{128}{3}$ cm^3

7 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is R = x(15 − x) and the cost is C = 7 + 9x.
(a) Show that the profit P = R − C is $P = -x^2 + 6x - 7$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 15x - x^2 - (7 + 9x) = -x^2 + 6x - 7$
$\frac{dP}{dx} = -2x + 6 = 0$ gives x = 3
$\frac{d^2P}{dx^2} = -2 < 0$, so a maximum
P = 2
Answer: 3000 items, maximum profit £2000

8 Percentages of percentages

In a group of students, 30% are boys and the rest are girls.
35% of the boys and 45% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 30 are boys and 70 are girls
35% of 30 = 10.5 and 45% of 70 = 31.5
10.5 + 31.5 = 42 out of 100
Answer: 42%

9 Estimating the median from grouped data

The table gives information about 106 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 26, 51, 61, 81, 106
The median is the value at position 106 ÷ 2 = 53
It is in the class 50 to 70, with 51 values before it, so we need 2 more of the 10 values in this class
Median ≈ $50 + \frac{2}{10} \times 20 = 54$
Answer: 54

10 Variation: percentage change

y is directly proportional to the cube of x.
x is decreased by 50%.
Work out the percentage change in y.

x is multiplied by 0.5
y is multiplied by 0.5^3
Multiplier for y = 0.125
Percentage decrease = 87.5%
Answer: 87.5% decrease