Day 26 **IGCSE 10-a-day: Model Solutions**

1 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length of the chord AB.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM^2 = OA^2 - OM^2 = 17^2 - 15^2 = 64$
AM = 8 cm
AB = 2 × 8 = 16 cm

(b) sin(angle AOM) = $\frac{8}{17}$

Angle AOM = 28.07°, so angle AOB = 2 × 28.07 = 56.1°
Answer: (a) 16 cm (b) 56.1°

2 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = 5x^4 + 6x + 5$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$
$5x^4 \to 20x^3$, $6x \to 6$
$\frac{dy}{dx} = 20x^3 + 6$

Answer: $\frac{dy}{dx} = 20x^3 + 6$

3 Multiplying 2 by 2 matrices

$A = \begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & -1 \\ -4 & -2 \end{pmatrix}$.
(a) Work out AB.
(b) Work out BA.

(a) AB: row 1 is 1 × (−2) + 1 × (−4) and 1 × (−1) + 1 × (−2); row 2 is 2 × (−2) + 5 × (−4) and 2 × (−1) + 5 × (−2)
$AB = \begin{pmatrix} -6 & -3 \\ -24 & -12 \end{pmatrix}$
(b) $BA = \begin{pmatrix} -4 & -7 \\ -8 & -14 \end{pmatrix}$
Answer: (a) $AB = \begin{pmatrix} -6 & -3 \\ -24 & -12 \end{pmatrix}$ (b) $BA = \begin{pmatrix} -4 & -7 \\ -8 & -14 \end{pmatrix}$

4 Inverse proportion in context

24 machines can make a batch of bolts in 12 hours.
After 6 hours, 8 of the machines leave. The rest carry on working at the same rate.
How many more hours does it take to finish?

Total work = 24 × 12 = 288 machine-hours
Work done = 24 × 6 = 144, so work left = 144
Workers left = 24 − 8 = 16
Time = 144 ÷ 16 = 9 hours
Answer: 9 hours

5 Maximum revenue

A company sells x thousand items each month. The price of each item is £32 − 2x.
The monthly revenue is £R thousand.
(a) Show that $R = 32x - 2x^2$.
(b) Find the maximum monthly revenue, showing that it is a maximum.

$R = x \times (32 - 2x) = 32x - 2x^2$

$\frac{dR}{dx} = -4x + 32 = 0$ gives $x = 8$

$\frac{d^2R}{dx^2} = -4 < 0$, so a maximum

$R = 32 \times 8 - 2 \times 8^2 = 128$

Answer: £128 thousand

6 Upper and lower bounds

a = 0.49 and b = 0.48,
both correct to 2 decimal places.
Work out the upper bound of a − b.

Each value is rounded, so it can be up to 0.005 above or below
$0.485 \leq a < 0.495$; $0.475 \leq b < 0.485$
Biggest answer: biggest a minus smallest b = 0.495 − 0.475 = 0.02
Answer: 0.02

7 Algebraic proof

Show algebraically that the product of any two consecutive even numbers is a multiple of 8.

Let the even numbers be 2m and 2m + 2, where m is an integer.
$2m(2m + 2) = 4m^2 + 4m = 4m(m + 1)$
m and m + 1 are consecutive integers, so one of them is even, and m(m + 1) = 2 × an integer
So 4m(m + 1) = 8 × an integer, a multiple of 8
Answer: Proved

8 Intersection and gradient

The curve $y = x^2 - 2x - 3$ and the line $y = -5x - 3$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$x^2 - 2x - 3 = -5x - 3$ gives $x^2 + 3x = 0$, so $x(x + 3) = 0$
$x = -3$ or $x = 0$, giving A (−3, 12) and B (0, −3)
$\frac{dy}{dx} = 2x - 2$: at A it is −8, at B it is −2
Answer: (a) A (−3, 12), B (0, −3) (b) −8 at A, −2 at B

9 Bounds in calculations

A metal block has mass 460 g, correct to the nearest 10 g, and volume 285 cm^3, correct to the nearest 5 cm^3.
Work out the upper bound of the density of the metal, in g/cm^3. Give your answer to 3 significant figures. (Calculator)

Mass: $455 \leq m < 465$. Volume: $282.5 \leq V < 287.5$
Density = mass ÷ volume
UB density = UB mass ÷ LB volume = 465 ÷ 282.5 = 1.65
Answer: 1.65 g/cm^3

10 Recasting solids

A solid cone has base radius 2 cm and height 27 cm.
It is melted down and recast into a solid sphere.
Work out the radius of the sphere.

The volume of a cone is $\frac{1}{3}\pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$.

Volume of cone = $\frac{1}{3} \times \pi \times 2^2 \times 27 = 36\pi$

$\frac{4}{3}\pi R^3 = 36\pi$, so $R^3 = 36 \times \frac{3}{4} = 27$

R = 3 cm

Answer: 3 cm