Day 259 **IGCSE 10-a-day: Model Solutions**

1 Laws of indices

Given that $\frac{25^3 \times 25}{5^3} = 5^n$,

find the value of n.

$25 = 5^2$

Numerator: $5^6 \times 5^2 = 5^8$

Denominator: 5^3

Subtract the powers: $8 - 3 = 5$

Answer: n = 5

2 Inverse and repeated functions

f(x) = 3x + 2.

(a) Find $f^{-1}(11)$.

(b) Express ff(x) in the form px + q.

(a) Solve 3x + 2 = 11: 3x = 9

x = 3 so $f^{-1}(11) = 3$

(b) ff(x) = 3(3x + 2) + 2 = 9x + 8

Answer: (a) 3 (b) ff(x) = 9x + 8

3 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = 2t^2 + t + 4 + \frac{12}{t}$ (t > 0)

Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = 4t + 1 - 12t^{-2}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 4 + 24t^{-3}$

When t = 2: v = 6 and a = 7

Answer: (a) 6 m/s (b) 7 m/s^2

4 Area and quadratic equations

The diagram shows a rectangle of length (x + 3) cm, width (2x + 2) cm and area 96 cm^2.

(a) Show that $2x^2 + 8x - 90 = 0$.

(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) (x + 3)(2x + 2) = 96

Expand: $2x^2 + 8x + 6 = 96$, so $2x^2 + 8x - 90 = 0$

(b) Factorise: $2(x - 5)(x + 9) = 0$

x = 5 (x = −9 is not possible as a length would be negative)

Length = 8 cm and width = 12 cm

Diagonal = $\sqrt{8^2 + 12^2} = \sqrt{208}$ = 14.4 cm (3 s.f.)

Answer: (b) 14.4 cm

5 Quadratic formula and surds

Solve $3x^2 + 6x - 5 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 3, b = 6, c = −5

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$x = \frac{-6 \pm \sqrt{96}}{6}$$

$\sqrt{96} = 4\sqrt{6}$, so $x = \frac{-6 \pm 4\sqrt{6}}{6}$

Divide the top and bottom by 2: $x = \frac{-3 \pm 2\sqrt{6}}{3}$

Answer: $\mathbf{x = \frac{-3 \pm 2\sqrt{6}}{3}}$

6 Differentiating brackets with square roots

$y = (\sqrt{x} + 2)^2$
Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x + 4x^{1/2} + 4$
$x \to 1$, $4x^{1/2} \to 2x^{-1/2}$
$\frac{dy}{dx} = 1 + 2x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = 1 + 2x^{-1/2}}$

7 Income tax

In one country, income tax is worked out like this:
The first £12,500 of income is not taxed.
The next £30,000 is taxed at 20%.
All income above that is taxed at 40%.
Nia earns £62,500 in a year.
Work out how much income tax Nia pays.

Taxed at 20%: £30,000
20% of £30,000 = £6,000
Taxed at 40%: £62,500 − £12,500 − £30,000 = £20,000
40% of £20,000 = £8,000
Total tax = £6,000 + £8,000 = £14,000
Answer: £14,000

8 Curve touching the x-axis

The curve $y = 2x^3 - 24x + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = 6x^2 - 24 = 0$ gives $x = -2$ or $x = 2$
The x-axis is a tangent when a turning point has y = 0
x = −2: 32 + k = 0, so k = −32
x = 2: −32 + k = 0, so k = 32
Answer: k = −32 or k = 32

9 Ratio and mixtures

Bottle A holds concentrate and water in the ratio 3 : 1.
Bottle B holds concentrate and water in the ratio 5 : 6.
8 litres from bottle A and 22 litres from bottle B are mixed.
Work out the ratio of concentrate to water in the mixture.

From A: concentrate = $8 \times \frac{3}{4} = 6$ litres, water = 2 litres

From B: concentrate = $22 \times \frac{5}{11} = 10$ litres, water = 12 litres

concentrate: 6 + 10 = 16 and water: 2 + 12 = 14
16 : 14 = 8 : 7
Answer: 8 : 7

10 Intersecting lines in a triangle

$\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OP} = \frac{2}{3}\mathbf{a}$ and $\mathbf{OQ} = \frac{3}{5}\mathbf{b}$.

AQ and BP meet at X.
(a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find the ratio AX : XQ.

Let $\mathbf{AX} = s\mathbf{AQ}$. Then $\mathbf{OX} = \mathbf{a} + s(-\mathbf{a} + \frac{3}{5}\mathbf{b}) = (1 - s)\mathbf{a} + \frac{3}{5}s\mathbf{b}$

Let $\mathbf{BX} = t\mathbf{BP}$. Then $\mathbf{OX} = \mathbf{b} + t(\frac{2}{3}\mathbf{a} - \mathbf{b}) = \frac{2}{3}t\mathbf{a} + (1 - t)\mathbf{b}$

Compare coefficients: $1 - s = \frac{2}{3}t$ and $\frac{3}{5}s = 1 - t$

Solving gives $s = \frac{5}{9}$ and $t = \frac{2}{3}$

(a) $\mathbf{OX} = \frac{4}{9}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) $AX = \frac{5}{9} \times AQ$, so $AX : XQ = \frac{5}{9} : \frac{4}{9} = 5 : 4$

Answer: (a) $\frac{4}{9}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) 5 : 4