**Day 258** **IGCSE 10-a-day: Model Solutions**

## 1 Tests of divisibility

The 4-digit number 198k is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 1 + 9 + 8 + k = 18 + k
This must be a multiple of 9, so k = 0 or 9
**Answer: k = 0 or 9**

## 2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 22°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 22°
Angle COB is an exterior angle of triangle OAC: 22 + 22 = 44°
(or: the angle at the centre is twice the angle at the circumference)
**Answer: (a) 90° (b) 44°**

## 3 Gradient where the curve crosses the x-axis

A curve has equation $y = (x + 1)(x - 2)(x + 3)$.
Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 + 2x^2 - 5x - 6$
$\frac{dy}{dx} = 3x^2 + 4x - 5$
The curve crosses the x-axis at $x = -3$, $-1$ and $2$
Gradients: $x = -3$ gives 10, $x = -1$ gives $-6$, $x = 2$ gives 15
**Answer: 10 at $x = -3$, $-6$ at $x = -1$, 15 at $x = 2$**

## 4 Inverse proportion in context

15 workers can build a wall in 30 days.
After 4 days, 2 of the workers leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 15 × 30 = 450 worker-days
Work done = 15 × 4 = 60, so work left = 390
Workers left = 15 − 2 = 13
Time = 390 ÷ 13 = 30 days
**Answer: 30 days**

## 5 Simplifying algebraic fractions

Simplify fully $\frac{x^2 - 49}{x^2 + 12x + 35}$

Factorise the top: $(x - 7)(x + 7)$
Factorise the bottom: $(x + 7)(x + 5)$
Cancel the common factor $(x + 7)$: $\frac{x - 7}{x + 5}$
**Answer:** $\frac{x - 7}{x + 5}$

## 6 Fractional power functions

$f(x) = -4x^{-1/2} - 4x^{-3/2}$

Find $f'(x)$ and work out the value of $f'(9)$.

$-4x^{-1/2} \to 2x^{-3/2}$, $-4x^{-3/2} \to 6x^{-5/2}$

$f'(x) = 2x^{-3/2} + 6x^{-5/2}$

$f'(9)$: $\sqrt{9} = 3$, so $f'(9) = \frac{8}{81}$

**Answer: $f'(x) = 2x^{-3/2} + 6x^{-5/2}$, $f'(9) = \frac{8}{81}$**

## 7 Solving AX = B

$A = \begin{pmatrix} 1 & 2 \\ 5 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by $A^{-1}$)

$A^{-1} = -\frac{1}{8}\begin{pmatrix} 2 & -2 \\ -5 & 1 \end{pmatrix}$

$X = -\frac{1}{8}\begin{pmatrix} 2 & -2 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

**Answer: $X = \begin{pmatrix} \mathbf{0} \\ \mathbf{-1} \end{pmatrix}$**

## 8 Tangent parallel to a chord

The diagram shows part of the curve $y = -2x^2 + 4x - 2$. A and B are the points on the curve with x coordinates $-2$ and 2. C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = $(-2, -18)$ and B = $(2, -2)$

Gradient of AB = $\frac{-2 - (-18)}{2 - (-2)} = 4$

$\frac{dy}{dx} = -4x + 4 = 4$, so $x = 0$

$y = -2$

**Answer: C $(0, -2)$**

## 9 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 6 cm, BC = 13 cm, CD = 12 cm,
angle ABC = 60° and angle ACD = 87°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 6^2 + 13^2 - 2 \times 6 \times 13 \times \cos 60°$ so AC = 11.269 cm

Area ABC = ½ × 6 × 13 × sin 60° = 33.77

Area ACD = ½ × 11.269 × 12 × sin 87° = 67.52

Total area = 33.77 + 67.52 = 101

**Answer: 101 $cm^2$**

## 10 Tree diagrams and reverse probability

Machine A makes 40% of the bolts in a factory. Machine B makes the rest.
60% of the bolts from machine A are faulty.
30% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.4 × 0.6 + 0.6 × 0.3 = 0.24 + 0.18 = 0.42
(b) P(A and faulty) = 0.4 × 0.6 = 0.24

P(A | faulty) = 0.24 ÷ 0.42 = $\frac{4}{7}$

**Answer: (a) 0.42 (b) $\frac{4}{7}$**