Day 257 **IGCSE 10-a-day: Model Solutions**

1 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(2x - 10)°$
Angle PQR = $(3x + 10)°$
Angle PRS = $(4x + 26)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$4x + 26 = 5x$
$x = 26$
(b) Angle PRS = 130° so angle PRQ = $180° - 130° = 50°$
Answer: (a) x = 26 (b) 50°

2 Multiplying and dividing surds

Simplify fully $\frac{2\sqrt{112}}{3\sqrt{7}}$

$\sqrt{112} = 4\sqrt{7}$
$\frac{2\sqrt{112}}{3\sqrt{7}} = \frac{8\sqrt{7}}{3\sqrt{7}}$
$= \frac{8}{3}$

Answer: $\frac{8}{3}$

3 Composite functions in x

$f(x) = 2x + 5$ and $g(x) = x^2 - 6$.
(a) Find fg(x).
(b) Find gf(x).

(a) $fg(x) = f(x^2 - 6) = 2(x^2 - 6) + 5$
$= 2x^2 - 7$
(b) $gf(x) = g(2x + 5) = (2x + 5)^2 - 6$
$= 4x^2 + 20x + 19$
Answer: (a) $fg(x) = 2x^2 - 7$ (b) $gf(x) = 4x^2 + 20x + 19$

4 Inverse proportion in context

15 painters can paint a hall in 24 days.
After 4 days, 5 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $15 \times 24 = 360$ painter-days
Work done = $15 \times 4 = 60$, so work left = 300
Workers left = $15 - 5 = 10$
Time = $300 \div 10 = 30$ days
Answer: 30 days

5 Show that dy/dx = ...

$y = \frac{(x+3)^2}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{(x-3)(x+3)}{x^2}$.

(b) Find the gradient when $x = -2$.

Expand: $(x+3)^2 = x^2 + 6x + 9$, so $y = x + 6 + 9x^{-1}$

$\frac{dy}{dx} = 1 - \frac{9}{x^2} = \frac{x^2 - 9}{x^2} = \frac{(x-3)(x+3)}{x^2}$

When $x = -2$: $\frac{dy}{dx} = -\frac{5}{4}$

Answer: (b) $-\frac{5}{4}$

6 Reading a trigonometric graph

The diagram shows the graph of $y = a \cos bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 4, least value = −2
c = ½(4 + (−2)) = 1 and a = ½(4 − (−2)) = 3
One full cycle takes 360° (1 cycle between 0 and 360), so b = 360 ÷ 360 = 1
Answer: a = 3, b = 1, c = 1

7 Normal in the form ax + by + c = 0

The point A lies on the curve $y = x^2 - x + 3$ where $x = 2$.
Find the equation of the normal at A. Give your answer in the form ax + by + c = 0 where a, b and c are integers.

y = 5 at x = 2 and gradient of curve = 3

Gradient of normal = $-\frac{1}{3}$

$y - 5 = -\frac{1}{3}(x - 2)$

$y = -\frac{1}{3}x + \frac{17}{3}$

Multiply by 3: $x + 3y - 17 = 0$
Answer: x + 3y − 17 = 0

8 Area of a segment

The circle has centre O and radius 4 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 4^2 = 19.548$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 4 \times 4 \times \sin 140° = 5.1423$ cm^2

Segment = sector − triangle = 19.548 − 5.1423 = 14.405
= 14.4 cm^2 (3 s.f.)
Answer: 14.4 cm^2

9 Vertical distance between stationary points

The curve $y = x^3 + 3x^2 - 9x + 2$ has two stationary points, A and B.
Work out the vertical distance between A and B.

$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x+3)(x-1)$

$x = -3$ and $x = 1$

At $x = -3$, $y = 29$; at $x = 1$, $y = -3$

Difference = 29 − (−3) = 32
Answer: 32

10 Tangency conditions

The line $y = 3x + k$ is a tangent to the curve $y = 2x^2 + x + 2$.
Find the value of k.

Equate: $2x^2 + x + 2 = 3x + k$

Rearrange: $2x^2 - 2x + 2 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $4 - 4 \times 2 \times (2 - k) = 0$

$8k - 12 = 0$

$k = \frac{3}{2}$

Answer: $k = \frac{3}{2}$