Day 256 **IGCSE 10-a-day: Model Solutions**

1 Divide then differentiate

$y = \frac{-6x^6 + 5x^2 - 1}{x^3}$

Find $\frac{dy}{dx}$.

Divide each term by x^3: $y = -6x^3 + 5x^{-1} - x^{-3}$

$-6x^3 \to -18x^2$, $5x^{-1} \to -5x^{-2}$, $-x^{-3} \to 3x^{-4}$

$\frac{dy}{dx} = -18x^2 - 5x^{-2} + 3x^{-4}$

Answer: $\mathbf{\frac{dy}{dx} = -18x^2 - \frac{5}{x^2} + \frac{3}{x^4}}$

2 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is greater than 8.

(b) Given that the first number is even, work out the probability that the product is greater than 8.

(a) There are $6 \times 5 = 30$ equally likely outcomes, and 15 of them satisfy: the product is greater than 8

Probability = $\frac{1}{2}$

(b) 15 outcomes satisfy: the first number is even. Of these, 8 also satisfy: the product is greater than 8

Probability = $\frac{8}{15}$

Answer: (a) $\mathbf{\frac{1}{2}}$ **(b)** $\mathbf{\frac{8}{15}}$

3 Completing the square

(a) Write $2 - 6x - x^2$ in the form $a - (x + b)^2$

(b) Write down the maximum value of $2 - 6x - x^2$ and the value of x for which it occurs.

(a) $2 - 6x - x^2 = -(x^2 + 6x) + 2$

$x^2 + 6x = (x + 3)^2 - 9$

so $2 - 6x - x^2 = -(x + 3)^2 + 9 + 2 = 11 - (x + 3)^2$

(b) The square is never negative, so the maximum is 11 when $x = -3$

Answer: (a) $\mathbf{11 - (x + 3)^2}$ **(b) maximum 11 at** $\mathbf{x = -3}$

4 Area of a triangle using sine

Triangle ABC has AB = 4 cm and BC = 6 cm.

The area of triangle ABC is 6 cm^2.

Angle ABC is acute.

Work out the size of angle ABC.

Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$6 = \frac{1}{2} \times 4 \times 6 \times \sin B$

$\sin B = 6 \div 12 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is acute, so $B = 30°$

Answer: 30°

5 Quadratic inequalities

Solve $3x^2 - 9x - 30 < 0$

Divide by 3: $x^2 - 3x - 10 < 0$

Solve $x^2 - 3x - 10 = 0$: $(x + 2)(x - 5) = 0$, so $x = -2$ or $x = 5$

The graph is a U shape crossing the x-axis at -2 and 5; it is below the axis between the roots

$-2 < x < 5$

Answer: $-2 < x < 5$

6 Turning point after a transformation

The graph of $y = f(x)$ has a minimum point at $(-2, 9)$.

Write down the coordinates and the type of turning point on each graph.

(a) $y = -f(x)$

(b) $y = f(x) - 5$

(a) $y = -f(x)$: reflection in the x-axis turns a minimum into a maximum

$(-2, -9)$, a maximum

(b) $y = f(x) - 5$: y-coordinate decreases by 5

$(-2, 4)$, a minimum

Answer: (a) $(-2, -9)$ maximum (b) $(-2, 4)$ minimum

7 Given a stationary point, find k

The curve $y = x^3 + kx$ has a stationary point at $x = -3$.

(a) Find the value of k.

(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + k$

$x = -3$: $27 + k = 0$, so $k = -27$

$\frac{dy}{dx} = 3x^2 - 27 = 3(x + 3)(x - 3)$

Other root: $x = 3$

$y = -54$

Answer: (a) $k = -27$ (b) $(3, -54)$

8 Tree diagrams and reverse conditional probability

Machine A makes 50% of the items and machine B makes the rest.

6% of items from A are faulty and 8% of items from B are faulty.

An item is chosen at random.

(a) Work out the probability that the item is faulty.

(b) The item is faulty. Work out the probability that it came from machine A.

(a) $0.5 \times 0.06 = 0.03$ and $0.5 \times 0.08 = 0.04$

P(faulty) $= 0.03 + 0.04 = 0.07$

(b) $\frac{0.03}{0.07} = \frac{3}{7}$

Answer: (a) 0.07 (b) $\frac{3}{7}$

9 Curve touching the x-axis

The curve $y = -x^3 + 6x^2 - 9x + k$, where k is a constant, has the x-axis as a tangent.

Find the two possible values of k.

$\frac{dy}{dx} = -3x^2 + 12x - 9 = 0$ gives $x = 1$ or $x = 3$

The x-axis is a tangent when a turning point has $y = 0$

$x = 1$: $-4 + k = 0$, so $k = 4$

$x = 3$: $0 + k = 0$, so $k = 0$

Answer: $k = 4$ or $k = 0$

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10 Laws of indices

Write $\frac{4^{n-1} \times 8^n}{8^n}$ in the form 2^{an+c}

where a and c are integers.

$4 = 2^2$, $8 = 2^3$, $8 = 2^3$

$4^{n-1} = 2^{2(n-1)}$, $8^n = 2^{3(n)}$, $8^n = 2^{3n}$

Exponent: $2(n - 1) + 3(n) - 3n = 2n - 2$

Answer: a = 2, c = −2