Day 255 **IGCSE 10-a-day: Model Solutions**

1 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (3x + 10)°, angle ACB = (x + 34)° and angle CAQ = 64°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
x + 34 = 3x + 10
2x = 24, so x = 12
(b) Angle PAB = 46°
Angles on the straight line PAQ: angle BAC = 180 − 46 − 64 = 70°
Answer: (a) x = 12 (b) 70°

2 Quadratic formula and surds

Solve $x^2 - 8x - 3 = 0$
Give your answers in the form $a \pm \sqrt{b}$

Complete the square: $(x - 4)^2 - 16 - 3 = 0$
$(x - 4)^2 = 19$
$x - 4 = \pm\sqrt{19}$
$x = 4 \pm \sqrt{19}$
Answer: $x = 4 \pm \sqrt{19}$

3 Solve f(x) = g(x)

$f(x) = x^2 - 5x - 3$ and $g(x) = -3x + 5$.
Solve f(x) = g(x).

$x^2 - 5x - 3 = -3x + 5$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x = -2$ or $x = 4$
Answer: $x = -2$ or $x = 4$

4 Pressure

A concrete slab in the shape of a cuboid weighs 1140 N.
Its smallest face has area 300 cm^2. Its largest face has area 3,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 300 cm^2 = 0.03 m^2 and 3,000 cm^2 = 0.3 m^2
(a) Greatest pressure is on the smallest face: 1140 ÷ 0.03 = 38,000 N/m^2
(b) Least pressure is on the largest face: 1140 ÷ 0.3 = 3,800 N/m^2
Answer: (a) 38,000 N/m^2 (b) 3,800 N/m^2

5 Finding a percentage rate

Ivan invests £6,000 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £7,986.
Work out the annual percentage interest rate.

Multiplier for 3 years = £7,986 ÷ £6,000 = 1.331
Yearly multiplier = the cube root of 1.331 = 1.1
10% increase each year
Answer: 10%

6 Curve touching the x-axis

The curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant, has the x-axis as a tangent.
Find the two possible values of k.

$\frac{dy}{dx} = 3x^2 - 6x - 9 = 0$ gives $x = -1$ or $x = 3$

The x-axis is a tangent when a turning point has $y = 0$
$x = -1$: $5 + k = 0$, so $k = -5$
$x = 3$: $-27 + k = 0$, so $k = 27$
Answer: $k = -5$ or $k = 27$

7 Ratio and algebra

In a club, the ratio of the number of boys to the number of girls is 5 : 2.
Then 15 boys are removed and 5 girls are removed.
The ratio is now 2 : 1.
How many girls were there at the start?

Let the numbers at the start be 5k and 2k
$(5k - 15) : (2k - 5) = 2 : 1$, so $1(5k - 15) = 2(2k - 5)$
$5k - 15 = 4k - 10$
$1k = 5$, so $k = 5$
Number of girls at the start: $2 \times 5 = 10$
Answer: 10

8 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -t^3 + 12t^2 + 3t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -3t^2 + 24t + 3$

$a = \frac{dv}{dt} = -6t + 24$

Maximum v when $a = 0$: $-6t + 24 = 0$, so $t = 4$

$\frac{d^2v}{dt^2} = -6 < 0$, so v is a maximum

$v = 51$
Answer: 51 m/s when t = 4 s

9 Bounds of a compound expression

p = 8.6 and r = 4, each correct to 1 decimal place.
q = 2.06, correct to 2 decimal places.

Work out the upper bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom
Top: UB p − LB q = 8.65 − 2.055 = 6.595
Bottom: LB r = 3.95
6.595 ÷ 3.95 = 1.67
Answer: 1.67

10 Stationary point of y = ax + b/√x

A curve has equation $y = x + \frac{54}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 1 - 27x^{-3/2}$

$\frac{dy}{dx} = 0$: $27x^{-3/2} = 1$, so $x^{3/2} = 27$

$x = 27^{2/3} = 9$, and $y = 27$

$\frac{d^2y}{dx^2} = \frac{81}{2}x^{-5/2}$

At $x = 9$, $\frac{d^2y}{dx^2} = \frac{1}{6}$, which is positive, so a minimum

Answer: (9, 27), minimum