Day 254 **IGCSE 10-a-day: Model Solutions**

1 Rate of change of area or volume

The surface area, S cm^2, of a sphere of radius r cm is $S = 4\pi r^2$.

(a) Find $\frac{dS}{dr}$ when r = 2.

(b) Find the value of r when $\frac{dS}{dr} = 16\pi$.

$\frac{dS}{dr} = 8\pi r$

When r = 2: $\frac{dS}{dr} = 16\pi$

$8\pi r = 16\pi$, so 8r = 16 and r = 2 (lengths are positive)

Answer: (a) 16π (b) r = 2

2 Time zones

A flight leaves London at 09:45 (London time).
It takes 13 hours 40 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 09:45 + 13 h 40 min = 23:25
Sydney is 9 hours ahead of London, so add 9 hours
08:25 the next day

Answer: 08:25 the next day

3 Area of a triangle using sine

Triangle ABC has AB = 5 cm and BC = 6 cm.

The area of triangle ABC is $\frac{15\sqrt{2}}{2}$ cm^2.

Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$\frac{15\sqrt{2}}{2} = \frac{1}{2} \times 5 \times 6 \times \sin B$

$\sin B = \frac{15\sqrt{2}}{2} \div 15 = \frac{\sqrt{2}}{2}$

$\sin 45° = \frac{\sqrt{2}}{2}$, and B is acute, so B = 45°

Answer: 45°

4 Exponential graph from points

The diagram shows part of the graph of $y = ab^x$.
The curve passes through (0, 1) and (2, 4).
(a) Find the values of a and b. (b) Work out the value of y when x = 3.

(a) At x = 0: $y = a \times b^0 = a$, so a = 1
At x = 2: $1 \times b^2 = 4$ so $b^2 = 4$
b = 2
(b) $y = 1 \times 2^3 = 8$

Answer: (a) a = 1, b = 2 (b) 8

Day 254 **IGCSE 10-a-day: Model Solutions**

5 Recurring decimals

Write $0.70\dot{0}\dot{2}$ as a fraction in its simplest form.

Let x = 0.7002...
100x = 70.0202...
10,000x = 7002.0202...
Subtract: 9900x = 6932
$x = \frac{6932}{9900} = \frac{1733}{2475}$

Answer: $\mathbf{\frac{1733}{2475}}$

6 Direct proportion formulae

C is directly proportional to the cube of d.
When d = 1, C = 12.
(a) Find a formula for C in terms of d.
(b) Work out the value of C when d = 3.

(a) $C = kd^3$
$12 = k \times 1$, so $k = 12 \div 1 = 12$
$C = 12d^3$
(b) $C = 12 \times 27 = 324$
Answer: (a) $\mathbf{C = 12d^3}$ **(b) 324**

7 Increasing and decreasing intervals

$y = -x^3 + 3x^2 + 24x + 5$
Find the range of values of x for which y is decreasing.

$\frac{dy}{dx} = -3x^2 + 6x + 24 = -3(x + 2)(x - 4)$
$\frac{dy}{dx} = 0$ when $x = -2$ and $x = 4$
The graph of $\frac{dy}{dx}$ against x is a ∩-shaped quadratic
y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$
So y is decreasing when $x < -2$ or $x > 4$
Answer: $\mathbf{x < -2}$ **or** $\mathbf{x > 4}$

8 Probability with sets

$n(\xi) = 40$, $n(A) = 17$, $n(B) = 11$ and $n(A \cap B) = 5$.
An element of ξ is chosen at random.
(a) Find the probability that it is in neither A nor B.
(b) Given that it is in A, find the probability that it is not in B.

$n(A \cup B) = 17 + 11 - 5 = 23$
(a) $n(A \cup B)' = 40 - 23 = 17$, so $P = \frac{17}{40}$
(b) In A there are 17 elements, and $17 - 5 = 12$ of them are not in B
$P = \frac{12}{17}$

Answer: (a) $\mathbf{\frac{17}{40}}$ **(b)** $\mathbf{\frac{12}{17}}$

9 Geometric sequences

The 2nd term of a geometric sequence is 27.
The 5th term of the sequence is 1.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 1 \div 27 = \frac{1}{27}$

$r = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

(b) The 2nd term is $a \times r^1$: $a \times \frac{1}{3} = 27$

$a = 27 \div \frac{1}{3} = 81$

Answer: (a) $r = \frac{1}{3}$ (b) 81

10 Minimum distance with 1/t

A particle P moves on a straight line. At time t seconds ($t > 0$), its distance from a point O is s metres, where

$s = t^2 + \frac{54}{t}$

Show that P is at its minimum distance from O when $t = 3$, and find this minimum distance.

$s = t^2 + 54t^{-1}$, so $v = \frac{ds}{dt} = 2t - 54t^{-2}$

$v = 0$: $2t = \frac{54}{t^2}$, so $t^3 = 27$ and $t = 3$

$\frac{d^2s}{dt^2} = 2 + \frac{108}{t^3}$, which is 6 when $t = 3$, so a minimum

$s = 27$

Answer: 27 m