Day 253 **IGCSE 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.
0.32, 85%, 0.89, 50%

Convert each to a decimal: 0.32 = 0.32, 85% = 0.85, 0.89 = 0.89, 50% = 0.5
In order: 0.32, 50%, 85%, 0.89
Answer: 0.32, 50%, 85%, 0.89

2 Rate of cooling or growth

The temperature, T °C, of a metal bar t minutes after it is removed from a heat source is modelled by

$T = 25 + \frac{80}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 2.

(b) Find the value of t when the temperature is falling at 80 °C per minute.

$T = 25 + 80t^{-1}$, so $\frac{dT}{dt} = -80t^{-2}$

When t = 2: $\frac{dT}{dt} = -20$

$-\frac{80}{t^2} = -80$, so $t^2 = 1$ and t = 1

Answer: (a) −20 °C/min (b) t = 1

3 Quadratic formula and surds

Solve $4x^2 - 5x - 7 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 4, b = −5, c = −7

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 4 \times (-7)}}{2 \times 4}$$

$$x = \frac{5 \pm \sqrt{137}}{8}$$

Answer: $x = \frac{5 \pm \sqrt{137}}{8}$

4 Determinant and inverse

$M = \begin{pmatrix} 1 & 0 \\ 4 & -3 \end{pmatrix}$.

(a) Work out the determinant of M.

(b) Find M^{-1}.

(a) det M = 1 × (−3) − 0 × 4 = −3
(b) Swap the leading diagonal, change the signs of the other two entries, divide by the determinant

$$M^{-1} = -\frac{1}{3}\begin{pmatrix} -3 & 0 \\ -4 & 1 \end{pmatrix}$$

Answer: (a) −3 (b) $-\frac{1}{3}\begin{pmatrix} -3 & 0 \\ -4 & 1 \end{pmatrix}$

5 Relative speed

Jamal sets off at 10:00 and cycles at 8 km/h along a path.
Maya sets off from the same place at 10:30 and cycles at 18 km/h along the same path.
At what time does Maya catch up with Jamal?

When Maya starts, Jamal is 0.5 × 8 = 4 km ahead
Maya gains 18 − 8 = 10 km every hour
Time to catch up = 4 ÷ 10 = 0.4 hours = 24 minutes
10:30 + 24 minutes = 10:54
Answer: 10:54

6 Two numbers: maximise or minimise

The positive numbers x and y satisfy x + y = 9.

Find the greatest possible value of x^2y, showing that it is the greatest.

$y = 9 - x$, so $x^2y = -x^3 + 9x^2$

$\frac{d}{dx}(x^2y) = -3x^2 + 18x = 0$, so $x(18 - 3x) = 0$ and $x = 6$ $(x > 0)$

Second derivative $= -6x + 18 = -18 < 0$, so a maximum

$y = 3$, so the greatest value is $6^2 \times 3 = 108$

Answer: 108

7 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the times, in minutes, taken by 200 runners to finish a race.

(a) Find the median. (b) Find the interquartile range.

(c) How many runners took more than 50?

(a) Median at cumulative frequency 100: reading across gives 20

(b) Lower quartile at 50 gives 10, upper quartile at 150 gives 50

IQR = 50 − 10 = 40

(c) At 50 the cumulative frequency is 150, so 200 − 150 = 50

Answer: (a) 20 (b) 40 (c) 50

8 Volume and surface area of composite solids

A solid is made from a cylinder of height 6 cm and a hemisphere. Both have radius 9 cm.

Work out the total volume of the solid. Give your answer in terms of π.

Cylinder: $\pi \times 9^2 \times 6 = 486\pi$

Hemisphere: $\frac{1}{2} \times \frac{4}{3} \times \pi \times 9^3 = 486\pi$

Total $= 486\pi + 486\pi = 972\pi$ cm^3

Answer: 972π cm^3

9 Show that the gradient is always positive

A curve has equation $y = 2x^3 + x^2 + x$.

Show that y is an increasing function for all values of x.

$\frac{dy}{dx} = 6x^2 + 2x + 1$

Discriminant $= 2^2 - 4 \times 6 \times 1 = -20$

The discriminant is negative, so $\frac{dy}{dx} = 0$ has no real solutions

The coefficient of x^2 is positive, so $\frac{dy}{dx} > 0$ for all x

So y is increasing for all values of x

Answer: Discriminant $= -20 < 0$, so $dy/dx > 0$ for all x

10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.3.

If it rains, the probability that Ali is late for school is 0.5.

If it does not rain, the probability that Ali is late is 0.2.

(a) Work out the probability that Ali is late.

(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.3 × 0.5 + 0.7 × 0.2 = 0.15 + 0.14 = 0.29

(b) P(rain and late) = 0.15

P(rain | late) = 0.15 ÷ 0.29 = $\frac{15}{29}$

Answer: (a) 0.29 (b) $\frac{15}{29}$