Day 252 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = 2t^3 + 2t^2 + 5t + 2$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 4.

$v = \frac{ds}{dt} = 6t^2 + 4t + 5$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t + 4$

When t = 4: v = 117 and a = 52

Answer: (a) 117 m/s (b) 52 m/s^2

2 Laws of indices with algebra

Simplify fully $\frac{2x^6y^2 \times 3x^5y^4}{2x^3y^3}$

Give your answer in the form kx^ay^b

Numbers: $2 \times 3 \div 2 = 3$
Powers of x: $6 + 5 - 3 = 8$
Powers of y: $2 + 4 - 3 = 3$
$= 3x^8y^3$

Answer: $3x^8y^3$

3 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 3.
50% of the juniors and 30% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 80 students in total: 20 juniors and 60 seniors
50% of 20 = 10 and 30% of 60 = 18
Together: 28 out of 80
$28 \div 80 \times 100 = 35\%$

Answer: 35%

4 Exact trigonometric values

Work out the exact value of $\tan 60° \div \cos 30°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\tan 60° \div \cos 30° = \sqrt{3} \div \frac{\sqrt{3}}{2}$

$= 2$

Answer: 2

5 Coordinates and gradient at the y-intercept

$y = 3x^4 + 2x^3 - x + 5$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 5, so A is (0, 5)

$\frac{dy}{dx} = 12x^3 + 6x^2 - 1$

When x = 0, $\frac{dy}{dx} = -1$

Answer: A = (0, 5), gradient -1

6 Composite with an inverse function

f(x) = 2x + 4 and g(x) = $2x^2 - 4x + 4$.

(a) Find $f^{-1}g(x)$.

(b) Work out $f^{-1}g(2)$.

(a) $f^{-1}(x) = \frac{x - 4}{2}$

$f^{-1}g(x) = \frac{2x^2 - 4x + 4 - 4}{2} = \frac{2x^2 - 4x}{2}$

(b) g(2) = 4, so $f^{-1}g(2) = \frac{4 - 4}{2}$

Answer: (a) $\frac{2x^2 - 4x}{2}$ (b) 0

7 Index equations

Solve $3^{2x - 1} = 9^{2x + 3}$

Write both sides as powers of 3: $9 = 3^2$

Equate the powers: $2x - 1 = 4x + 6$

$-2x = 7$, so $x = -\frac{7}{2}$

Answer: $x = -\frac{7}{2}$

8 Percentage change and proportion

The volume of a cube is directly proportional to the cube of the length of its edge.

The length of its edge is increased by 10%.

Work out the percentage change in the volume of a cube.

The length of its edge is multiplied by 1.1

The multiplier for the volume of a cube is $1.1^3 = 1.331$

1.331 is an increase of 33.1%

Answer: 33.1% increase

9 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 14 cm.

The vertex V is vertically above the centre O of the base. VA = 18 cm.

(a) Work out the vertical height VO.

(b) Work out the angle between VA and the base. (Calculator)

(a) AC = $14\sqrt{2}$ so AO = 9.899 cm

$VO^2 = 18^2 - 9.899^2 = 226.0$

VO = 15.0 cm

(b) $\cos VAO = \frac{AO}{VA} = \frac{9.899}{18}$, angle = 56.6°

Answer: (a) 15.0 cm (b) 56.6°

10 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, −5) lies on the curve and the gradient of the curve at this point is 2.

Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$

Point on curve: $a + b = -5$. Gradient: $2a - b = 2$

Solving simultaneously gives $a = -1$ and $b = -4$

Answer: $a = -1$, $b = -4$