Day 251 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Differentiate $-5x^3 - 8x - 3$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$-5x^3 \to -15x^2$, $-8x \to -8$

$\frac{dy}{dx} = -15x^2 - 8$

Answer: $\mathbf{\frac{dy}{dx} = -15x^2 - 8}$

2 Matrix times vector and order

$C = \begin{pmatrix} -3 & -3 \end{pmatrix}$ and $A = \begin{pmatrix} 4 & 3 \\ 3 & -2 \end{pmatrix}$.

(a) Write down the order of matrix C.

(b) Work out CA.

(a) 1 row and 2 columns: 1 × 2

(b) $CA = \begin{pmatrix} (-3) \times 4 + (-3) \times 3 & (-3) \times 3 + (-3) \times (-2) \end{pmatrix}$

$CA = \begin{pmatrix} -21 & -3 \end{pmatrix}$

Answer: (a) 1 × 2 (b) $\mathbf{\begin{pmatrix} -21 & -3 \end{pmatrix}}$

3 Equation of a tangent

A curve has equation $y = -2x^3 + 3x + 6$.

Find the equation of the tangent to the curve at the point where $x = -3$.

When $x = -3$, $y = 51$

$\frac{dy}{dx} = -6x^2 + 3$, so the gradient at $x = -3$ is -51

$y - 51 = -51(x + 3)$

$y = -51x - 102$

Answer: $\mathbf{y = -51x - 102}$

4 Finding a percentage rate

A car was bought for £1,100.

Each year its value decreases by the same percentage.

After 2 years it is worth £176.

Work out the percentage decrease each year.

Multiplier for 2 years = £176 ÷ £1,100 = 0.16

Yearly multiplier = the square root of 0.16 = 0.4

60% decrease each year

Answer: 60%

5 Overall percentage change from repeated changes

The value of a machine falls by 10% each year for 2 years.

Work out the overall percentage decrease. Give your answer correct to 3 significant figures.

Multiplier = 1 − 10 ÷ 100 = 0.9

Overall multiplier = $0.9^2 = 0.81000$

Percentage decrease = (1 − 0.81000) × 100 = 19.0%

Answer: 19.0%

6 Venn diagrams with algebra

The Venn diagram shows how many of 45 students play the piano (P) or the guitar (G).
(a) Work out the value of x.
(b) A student is chosen at random from those who play the guitar. Work out the probability that this student also plays the piano.

(a) Total: $(x + 4) + x + (2x - 2) + 7 = 45$
$4x + 9 = 45$, so $x = 9$
(b) Students in the second circle: $x + 2x - 2 = 9 + 16 = 25$, of whom 9 are in both
Probability = $\frac{9}{25}$

Answer: (a) x = 9 (b) $\frac{9}{25}$

7 Percentages with an unknown rate

The price of a share is decreased by x%.
The new price is then increased by x%.
The final price is 36% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 36% less, so the multiplier is 0.64: $1 - \frac{x^2}{10000} = 0.64$

$\frac{x^2}{10000} = 0.36$, so $x^2 = 3600$ and $x = 60$
Answer: x = 60

8 Radius from chord and height of arc

A circular arc has a chord AB of length 30 cm.
The greatest distance between the chord and the arc is 3 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so $OM = r - 3$
$AM = 30 \div 2 = 15$. Pythagoras in triangle OMA: $r^2 = (r - 3)^2 + 15^2$
$r^2 = r^2 - 6r + 9 + 225$
$6r = 234$, so $r = 39$ cm
Answer: 39 cm

9 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 21) and (3, 189).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 21$ and $k\,a^3 = 189$
Divide: $a^2 = \frac{189}{21} = 9$
$a = 3$ (positive, since $a > 0$)
From the first point: $k \times 3 = 21$, so $k = 7$
Answer: k = 7, a = 3

10 Stationary points of a quartic

A curve has equation $y = 2x^4 - 16x^2 - 6$.

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 32x = 8x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 2$ or $x = -2$

$y = -6$ when $x = 0$, and $y = -38$ when $x = \pm 2$

$\frac{d^2y}{dx^2} = 24x^2 - 32$: it is -32 at $x = 0$ (maximum) and 64 at $x = \pm 2$ (minimum)

Answer: (0, −6) maximum; (2, −38) and (−2, −38) minima