Day 250 **IGCSE 10-a-day: Model Solutions**

1 Composite functions

$f(x) = -3x - 7$ and $g(x) = x^2 - 1$

(a) Find $gf(x)$. Give your answer in the form $ax^2 + bx + c$.

(b) Solve $gf(x) = 255$

(a) $gf(x) = g(-3x - 7) = (-3x - 7)^2 - 1$

$= 9x^2 + 42x + 48$

(b) $(-3x - 7)^2 = 256$, so $-3x - 7 = \pm 16$

$-3x - 7 = 16$ or $-3x - 7 = -16$

$x = -\frac{23}{3}$ or $x = 3$

Answer: (a) $9x^2 + 42x + 48$ (b) $x = -\frac{23}{3}$ or $x = 3$

2 Tangent meets the axes

The diagram shows the curve $y = -x^2 + 3x + 1$ and the tangent at P(1, 3), meeting the axes at A and B.

(a) Find an equation of the tangent at P.

(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = -2x + 3$, so at $x = 1$ the gradient is 1

$y - 3 = 1(x - 1)$, so $y = x + 2$

B: $x = 0$ gives $y = 2$. A: $y = 0$ gives $x = -2$

Area $= \frac{1}{2} \times 2 \times 2 = 2$

Answer: (a) $y = x + 2$ (b) 2

3 Rationalising the denominator

Rationalise the denominator of $\frac{11}{3\sqrt{7}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$:

$\frac{11}{3\sqrt{7}} = \frac{11\sqrt{7}}{21}$

Answer: $\frac{11\sqrt{7}}{21}$

4 Tangent at a turning point

The graph of $y = -2x^2 + 4x + 6$ has a turning point at P.

Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -4x + 4$

At P, $\frac{dy}{dx} = 0$, so $-4x + 4 = 0$ and $x = 1$

$y = 8$ at P

The gradient is 0, so the tangent is horizontal: $y = 8$

Answer: $y = 8$

5 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).

The equation of the line of best fit is $y = -1.5x + 24$.

(a) Estimate y when $x = 3$.

(b) Explain why an estimate of y when $x = 20$ may be unreliable.

(a) $y = -1.5 \times 3 + 24 = 19.5$

(b) $x = 20$ is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

Answer: (a) 19.5 (b) x = 20 is outside the range of the data

6 Geometric sequences

A geometric sequence has term 3 equal to 1 and term 6 equal to $-\frac{1}{8}$.

Find the common ratio and the first term.

Term 3 = a r^2, term 6 = a r^5

Divide: $r^3 = -\frac{1}{8} \div 1 = -\frac{1}{8}$

$r = -\frac{1}{2}$

$a = 1 \div (-\frac{1}{2})^2 = 4$

Answer: r = $-\frac{1}{2}$, first term = 4

7 Tangent to a circle

A circle has equation $x^2 + y^2 = 125$.
The point P(−10, −5) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{-5-0}{-10-0} = \frac{1}{2}$

The tangent is perpendicular to the radius, so its gradient is −2. Using P: y + 5 = −2(x + 10)
Expand and rearrange: 2x + y = −25
Answer: 2x + y = −25

8 Transformations of graphs

The graph of y = f(x) has a minimum point at (−1, 4).
Write down the coordinates of the turning point of the graph of
(a) y = f(x + 4)
(b) y = f(−x) + 1

(a) Start with the turning point (−1, 4)
Apply the transformation to the point: (−5, 4)
(b) f(−x) changes the x-coordinate to 1, and f(x) + 1 changes the y-coordinate to 5
Turning point: (1, 5)
Answer: (a) (−5, 4) (b) (1, 5)

9 Area of triangle from a tangent

The tangent to the curve $y = 3x^2 + 2x + 3$ at the point where x = −3 meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

y = 24 and $\frac{dy}{dx} = 6x + 2$, so the gradient is −16

Tangent: y = −16x − 24

y-axis: (0, −24). x-axis: 0 = −16x − 24 so $x = -\frac{3}{2}$

Area = $\frac{1}{2} \times 24 \times \frac{3}{2}$

= 18

Answer: 18 square units

10 Bounds of a compound expression

p = 13.5 and r = 5.1, each correct to 1 decimal place.
q = 1.58, correct to 2 decimal places.

Work out the lower bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom
Top: LB p − UB q = 13.45 − 1.585 = 11.865
Bottom: UB r = 5.15
11.865 ÷ 5.15 = 2.30
Answer: 2.30